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Anisotropy Problem in Geostatistical Simulation by Orthogonal Transformed Indicator Methods

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ABSTRACT: This paper explores the manner in which orthogonal transformed indicator methods (OTIM) handle with anisotropy in stochastic simulation. In orthogonal transform, three decomposition algorithms are considered: Spectral, Symmetric and Cholesky-Spectral. Using a simulated deposit with anisotropy ratio 1/3, all three algorithms are evaluated in terms of grade variogram and grade-tonnage curves reproduction.

1 INTRODUCTION

Conditional cumulative distribution function (ccdf) plays an important role in geostatistical estimation and sequential simulation. Indeed, data-dependent optimal estimations are computed from conditional distribution functions and sequential simulations are obtained randomly drawing from conditional distribution functions. A variety of method for estimating conditional distribution functions is suggested. These are classified as parametric and nonparametric. This study İs concerned with nonparametric approach, especially orthogonal transformed indicator method of this approach. The conditional distribution functions and their nonparametric estimation are described in Goovaerts (1997) and Tercan and Kaynak (1999).

Orthogonal transformed indicator method (Tercan, 1999) İs a compromise between the two extremes of indicator cokriging and indicator kriging. It requires less estimation and modelling over indicator cokriging and uses more information over indicator kriging. The idea behind this approach is to transform the indicator functions into a set of spatially orthogonal functions (factors) and to use the autokrigeability property of these functions. Orthogonalization of indicator function relies on principally the decomposition of the indicator variogram matrices as a matrix product.

Despite the aforementioned advantages of OTIM, the approach may be problematic when the variable studied reveals an anisotropic structure. Indeed, it İs not possible to construct the indicator variogram matrices anisotropically because they are estimated either omnidirectionally or in a particular direction. The purpose of this study is to investigate how the conditional distribution functions obtained using orthogonal transformed indicator methods work in geostatistical simulation in the presence of anisotropy. In the estimation of conditional distribution function, three decomposition algorithms are considered; Spectral (SPEC), Symmetric (SYMM) and Cholesky-Spectral (CHSP) decomposition. For comparison, indicator knging (INDI) is also used.

Geostatistical simulations are mainly used for generation of equi-probable alternative realizations of mineral grade and geologic features with specified histogram and variogram. Typically, these realizations are fed into a transfer function developed as a logical equivalent of mineral deposit. By processing multiple equiprobable realizations through the transfer function, an equivalent number of responses are obtained, i.e. a response distribution. This distribution of responses provides a probabilistic assessment of the uncertainty associated with the input variable (Jourael, 1989). In the present study, the decomposition algorithms are evaluated in this setting. First of all, the ability of the decomposition algorithms in reproducing anisotropic variograms is examined. Two transfer functions are defined; proportion and average of grade values above a specific cutoff, yielding grade-tonnage curves as responses. The uncertainty of grade tonnage curves is assessed by the distribution of responses for each algorithms.

2 SEQUENTIAL SIMULATION

Consider the simulation of variable grade Z at N grid nodes x_n conditional to the data set [z(Xa), ct=l,...,n].

Sequential simulation (Journel and Alabert, 1988; Gomez-Hernandez and Srivastava, 1990) amounts to modelling the conditional distribution function then sampling it at each of the grid nodes visited along a random sequence. When a nonparametric approach is considered an indicator-based method is used. To ensure reproduction of the grade variogram model, each ccdf is made conditional not only to the original n data but also to all values simulated at previously visited locations. Multiple realizations are obtained by repeating the entire sequential drawing process Sequential simulation starts with the transform of an indicator vector into the spatially orthogonal factors.

3 CASE STUDY

Figure 1 shows the spatial distribution of 2500 conditionally simulated values on a 50 \ 50 m regular grid at a level of Kure (Asikoy) copper deposit and hereafter considered as reference values for subsequent work.

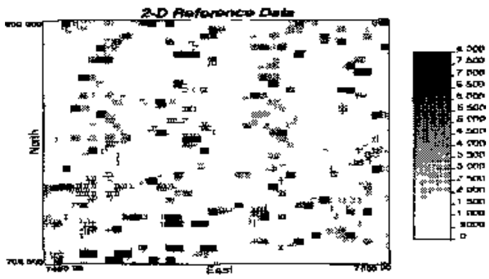

Figure 1 The spatial distribution of reference data sel

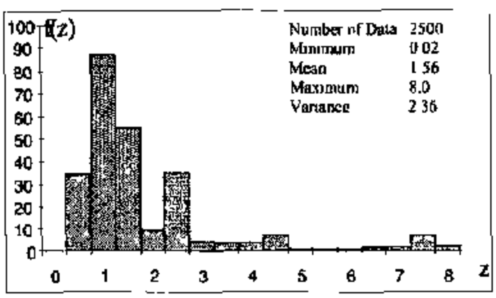

Figure 2 A frequency distribution of reference data set

The simulated annealing algorithm given in GSLIB (Deutsch and Journel, 1998) was used to force the realization to match an anisotropic spherical vanogram with nugget effect 0.3, partial sill 2.1 and range 38 m in NS direction and 12 m m EW direction. Figure 3 and 4 show frequency distribution and variograms of the reference data set.

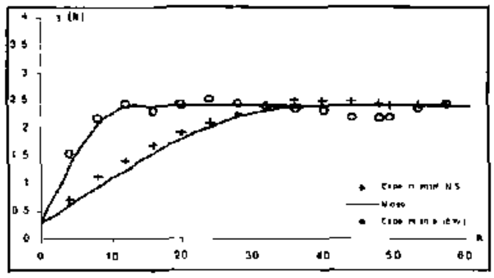

Figure 3 The experimental and input vanograms of the reference data set m the NS and EW directions

The purpose of this study is to evaluate the decomposition algorithms in the presence of anisotropy under perfect conditions. So the problem of statistical inference of the variogram functions will not be addressed here Instead, the variogram models deduced from reference (2500 data) information are used.

As there are no economic and technical restrictions, the nine cutoff values corresponding to the nine deciles of the reference distribution are used: these are; 0.46, 0.60, 0.73, 0 82, 1.10, 1.20, 1.44,2.15 and3.21.

Factor variograms were computed for nine cutoff values. As the order of the CHSP and SPEC factors gets higher, the range of spatial correlation decreases and essentially vanishes for the sixth factor in the NS direction and for the third factor in the EW direction. However, the variogram of the SYMM factors does not display any decrease in spatial correlation. All the factor variograms with a spatial correlation were modelled with a geometric anisotropy model with a larger range in the NS direction (these variograms are not shown here).

One hundred realisations of grade were generated using each of the three decomposition algorithms and also indicator simulation. The first realizations are shown in Figure 4. SISIM given in Deutsch & Joumel (1998) is modified in order to handle with OTTM.

Figure 5a-b show experimental variograms of the 100 realizations for each of algorithms. These figures indicate that there are large discrepancies (fluctuations) between input and realization variograms for all algorithms. This is an expected result since simulation from an indicator-derived (either indicator or orthogonal transformed indicator kriging) ccdf guarantees reproduction of indicator variogram for the cutoffs Zk considered not the grade variogram. In theory, the reproduction of the grade variogram is guaranteed only if indicator cokriging with infinite number of cut off values are used as pointed out by Armstrong & Dowd (1994) and Deutch & Joumel (1998).

Neither sequential orthogonal transformed indicator simulation (SOTIS) nor sequential indicator simulation (SIS) seems to reproduce anisotropy well (Figure 5). This may be related to smaller number of grid nodes (2500) and smaller size of simulation area respect to the longer range of input anisotropic variogram model (ratio being 1/5 only). However, one obvious result is that SOTIS works as good as SIS in reproduction of anisotropic variogram.

Also note that realization variograms have higher nugget effect than the input variogram model. This high nugget is a result of the discretization procedure used in indicator approach. Indeed nonparametric techniques are applied at K discretization cutoffs Zk and they provide ccdf s for these cutoff values only. In stochastic simulation, it is necessary to complete ccdf for all values other than K. discretization cutoffs z_k The conditional distribution functions are completed by interpolating between ccdf values and extrapolating beyond them. However, the interpolation / extrapolation of ccdf values is done independently from one location to another. This makes simulated grade values within the same class (zu, Zk) spatially uncorrected. Consequently, whatever the number of cutoffs, the sequential realizations have high nugget variances as a result of this artificial noise within classes. OTIM is no exception and shares the same property.

Estimated conditional distribution function may not satisfy the order relations of a valid distribution function. For example, cdf value may be less than 0 or greater than 1 or they may be decreasing with increasing cutoff values. When order relations violations occur, they must be corrected. These corrections for order relations do not impact the reproduction of the grade variogram but the indicator variogram. Indeed experience shows that the SYMM algorithm produces significiantly less order relations than other algorithms but the SYMM fluctuations do not differ much from those of other algorithms.

3.1 *The response distributions*

The proportion and average of the values above the nine cutoffs corresponding to the nine deciles of the reference distribution are calculated first for the reference data set and then for each realization. Figure 6 shows the response distnbution for the proportion (tonnage) while Figure 6 presents the response distribution for the average (mean grade) for SOTIS and also SIS. In these figures, the response distributions at each cutoff are presented using box-plots. In addition, the true proportion and

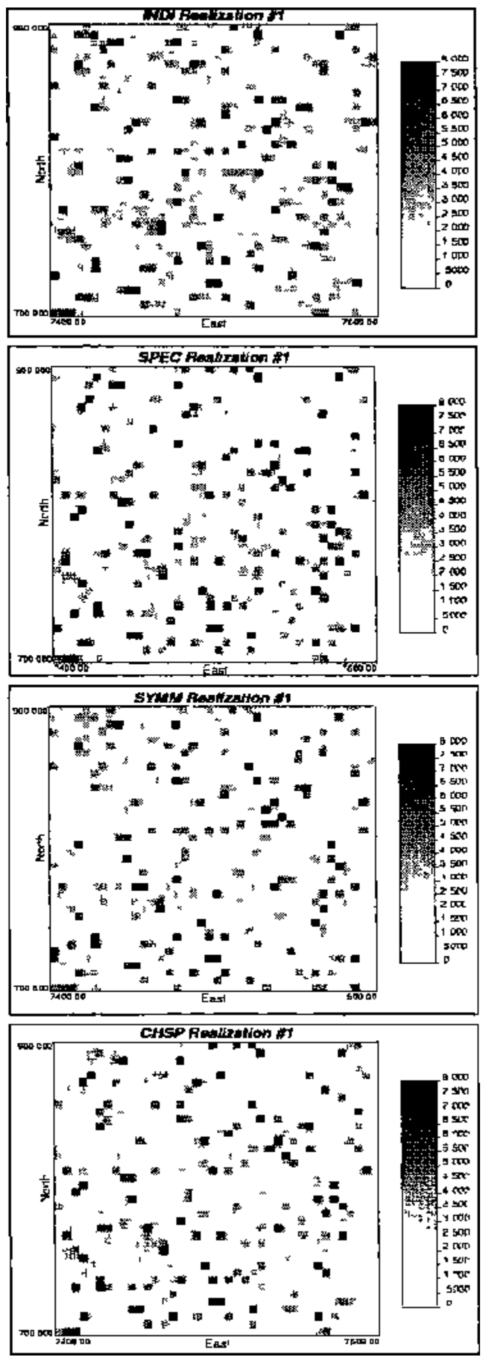

Figure 4 First realizanon of ihe spatial distribuions generated by three decomposition algorithms and indicator kngmg

average values calculated from the reference distribution are marked with ' * ' in these figures.

AH the algorithms can be said to be accurate since the response distibutions of both proportion and average contain the true values. Common to all the decomposition algorithms and indicator approach is an increase in variability of the averages with increasing cutoff. However, for the distribution of the proportion, the increase is seen only at and around median cutoff (1.1). Above all, all the algorithms yield similar uncertainty distributions.

One reason for this is the high number of conditioning data (2500) used in simulation. As this number decreases, one may expect largest differences between algorithms.

4 CONCLUSIONS

Although the conclusions that can be drawn from this study are specific to the data set studied, it is clear that sequential simulations based on orthogonal transformed indicator method reproduce anisotropic variograms as good as sequential indicator simulation algorithm. There seems to be no big difference between the decomposition algorithms when considering the distribution of grade tonnage curves. This is due to the high number of conditioning data. A similar study should be done with no conditioning data in order to see the differences between algorithms.

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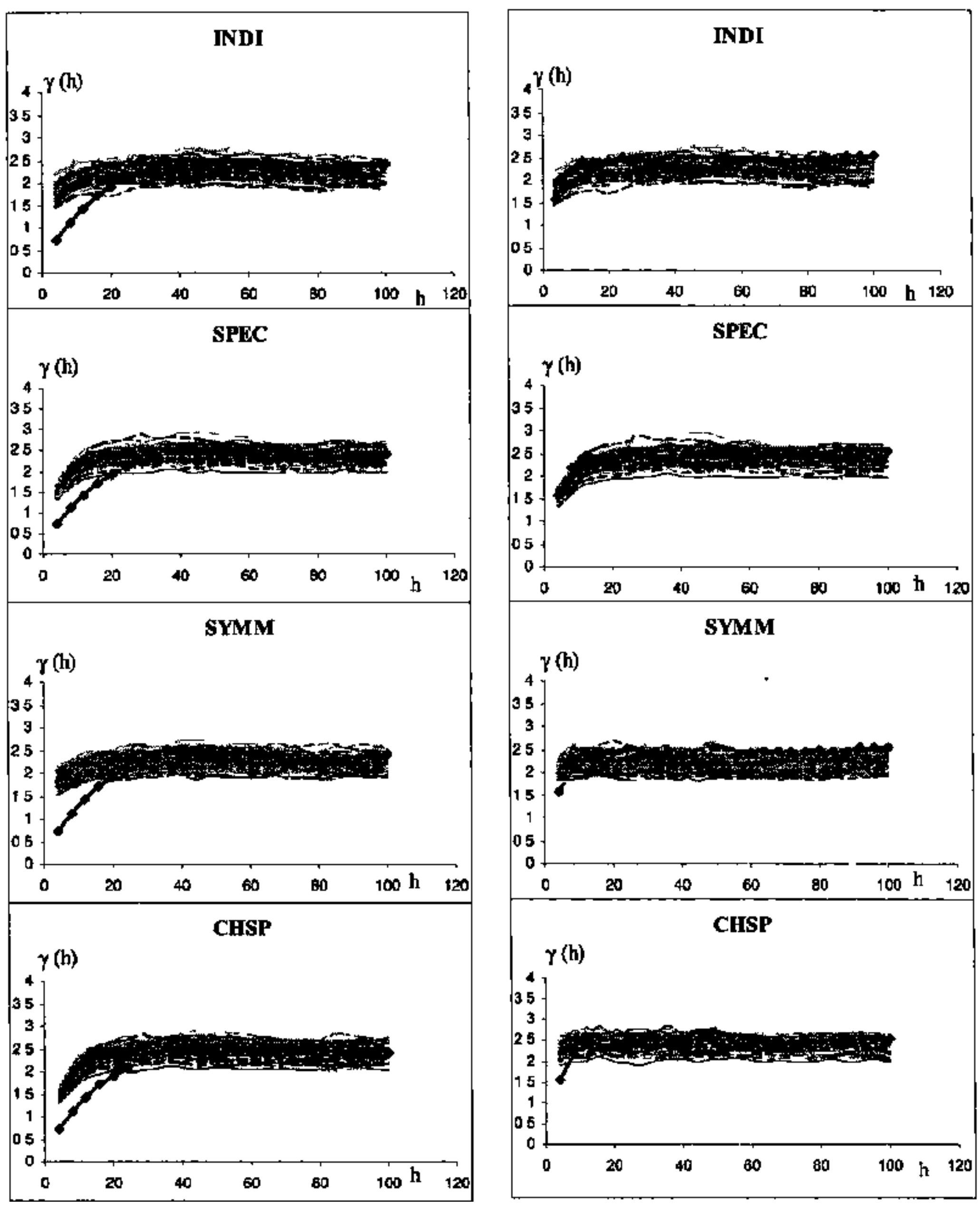

H *(b)*

Figure 5 (a) Vanograms of the realizations for INDI, SPEC, SYMM and CHSP in NS direction, (b) Vanograms of the realizations for INDI, SPEC, SYMM and CHSP in EW direction

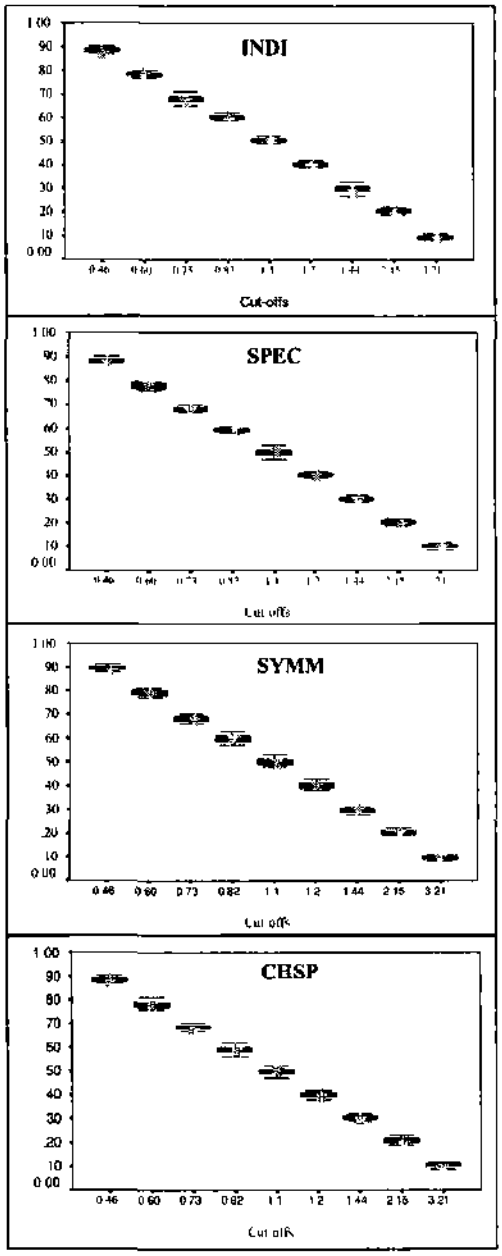

Figure 6 The uncenainity distribution for the proportion

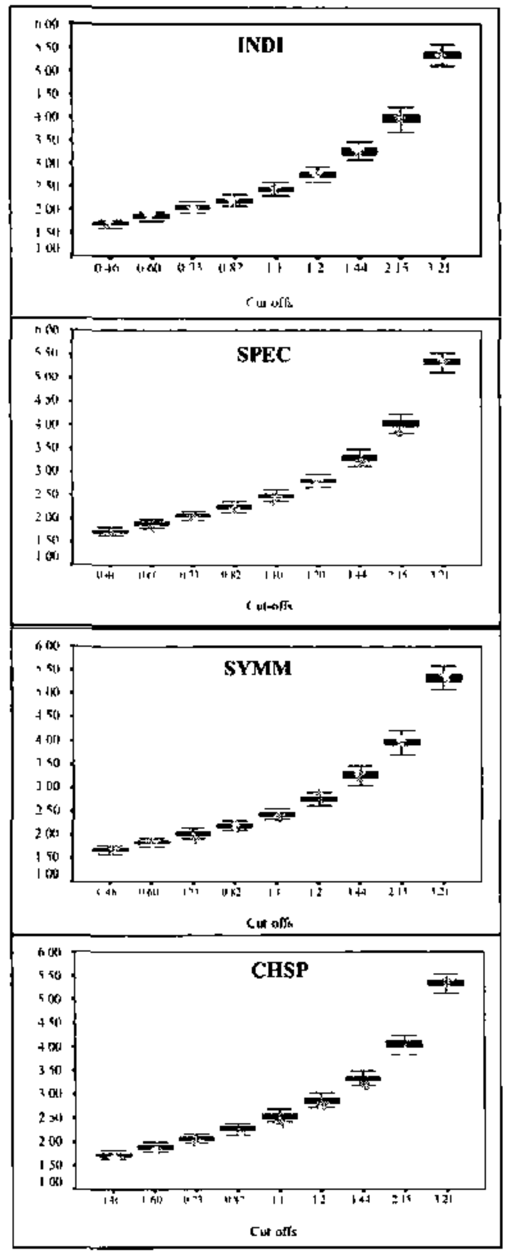

Figure 7 The uncertamity distribution for the a\erage