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Geostatistical Simulation of a Quarry

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ABSTRACT: Mine production scheduling requires quantification of the variability of the attributes of the mined product as delivered to the facility. Geostatistical simulation is a method of generating, on any specified scale, realisations of these attributes. Geostatistical simulation provides a set of values that can be used in mine production planning. In this paper, after the importance of geostatistical simulation is briefly reviewed, the sequential Gaussian simulation, which has been widely accepted for the simulation of in-situ mineral properties, is introduced in a case study.

I INTRODUCTION

Data on the in-situ characteristics of the mineral product are obtained from drilling and other sampling programmes such as grade control and blast hole sampling. The grades of planning blocks from the grades of drill holes and/or the grades of blast hole cuttings are commonly estimated by linear kriging (e.g., simple or ordinary kriging). These estimators, based on the least squares method, have significant drawbacks:

1. They are conditionally biased because they implicitly assume a cost or loss function that equally penalises underestimation and overestimation. Unless a Gaussian model for errors is assumed, linear kriging methods yield an inadequate measure of local accuracy.
2. Linear kriging methods yield smoothed results, which cannot be used for these applications because of their sensitivity to the existence of extreme values and their patterns of continuity. Geostatistical simulation can assess uncertainty in production scheduling by use as a "transfer function". To put it another way, risk arising from simulation is assessed by multiple realisations.
3. Linear kriging methods can estimate on the basis of fixed support (e.g., point or block). As the size of the support changes, it is necessary to use cumbersome and assumption-dependant correction methods.

Conditionally unbiased methods are, in general, non-linear. These methods do, however, require significantly more assumptions, which are often unverifiable, and they can be prohibitively time-consuming. Furthermore, these methods provide a sense only of local uncertainty because each conditional cumulative distribution function deals with a single location. Notice that single point ccdfs (cumulative conditional distribution functions) do not ensure the quantification of spatial uncertainty. For example, the prediction of grade fluctuations for various mining and processing decisions (e.g., extraction method, production schedules, milling and stockpiling) requires the assessment of spatial uncertainty rather than that of local uncertainty.

2 GEOSTATISTICAL SIMULATION

Geostatistical simulation provides a set of values that conform to the following criteria (Dowd, 1993):

(i) At all sampled locations they coincide with the actual values,
(ii) They have the same spatial dispersion, i.e., same variogram, as the true values,
(iii) They have the same distribution as the true values.
(iv) They are co-regionalized with any other simulated variable in the same way as the true values.

A set of values conforming to these criteria is called a conditional simulation, i.e., a simulation that is conditional on the simulated values coinciding with the true values at the sampled locations. A non-conditional simulation has attributes (Ü), (iii) and (iv), but not (i).

Conditional simulation does not create data; it simply provides one possibility (among an infinite

number) of what may actually be present at non-sampled locations. This approach amounts to considering the true values as one particular realisation of a random function; each conditional simulation then provides another.

Estimation and simulation are two separate and distinct procedures with different objectives and different results. The objective of estimation is to provide the best (however defined) estimate of a variable at any location. The objective of simulation is to provide a set of values that conform to the criteria listed above, i.e., values that reproduce the characteristics, or behaviour, of the phenomenon as observed in the available data.

Stochastic simulation methods are used:

a. To assess the impact of uncertainty. Stoc&astic simulation provides a means of assessing risk (Dowd, 1994 and Goovaerts, 1999). In this kind of study, many alternate models are generated and processed to construct a distribution of possible values for specified attributes. This distribution is used to evaluate the risk associated with the uncertainty at unsampled locations- Simulation models can also be used for decision-making under uncertainty.
b. To honour heterogeneity. Stochastic simulation reproduces spatial variability. In some cases, only one outcome is used as a basis for performance prediction. Stochastic simulation methods enhance the ability to produce a realistic level of heterogeneity.
c. To obtain complex information. Geostatistical simulation can incorporate an increasingly broad range of information that cannot be accommodated by more conventional methods.

Figure 1 illustrates differences between the use of linear kriging and geostatistical simulation. Geostatistical simulation can be used to assess uncertainty in the production scheduling. The estimation of block model, a posterior scheduling is evaluated as a "transfer function" (Rossi, 1998). Geostatistical simulation provides a response distribution by generating multiple images, i.e., a series of schedules based on possible realisations of the block grades.

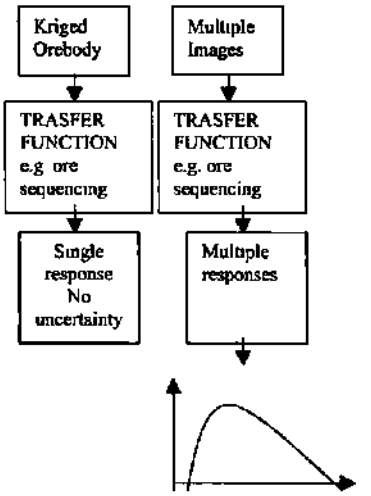

Figure 1 Comparison of linear krigmg and geostatistical simulation.

2.1 *Sequential Approach to Simulation*

The basis of sequential simulation is that conditioning is extended to include all data within a neighbourhood that includes the original data and all previously simulated values (Joumel, 1989). Sequential methods are based on an application of Bayes* Theorem:

$$P(A_1, A_2, \ldots, A_n) = P(A_n | A_1, \ldots, A_{n-1}) \cdot P(A_{n-1} | A_1, \ldots, A_{n-2}) \cdot \ldots \cdot P(A_2 | A_1) \cdot P(A_1) \quad (1)$$

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Given the joint simulation of z-values at *k* locations surrounded by *n* data, the realisations can be generated by drawing from a conditional cumulative distribution function:

$$F(u'_1, u'_2, \ldots, u'_k; z_1, z_2, \ldots, z_k | (n)) = F(u'_k; z_k | (n+k-1)) * \ldots * F(u'_2; z_2 | (n+1)) * F(u'_1; z_1 | (n)) \quad (2)$$

where $|(n+k-1)$ designates conditioning to the n data values and to the k-1 previous realisations. This decomposition makes it possible to generate a realisation of a random vector $\{Z(u'_j), j = 1, \ldots, K\}$ in *K* successive steps:

1. Determine the conditional distribution at the first location, which is conditional on the available *n* data:

$$F(u'_1; z | (n)) = Prob\{Z(u'_1) \leq z | (n)\} \quad (3)$$

2. Draw a value z; from the conditional distribution of Z; given the *n* data
3. Draw a value *Z2* from the conditional distribution of *Z2* given the n original data and that $Z_t = z_s$

K. Draw a value z_K from the conditional distribution of Z_K given the *n* original data and that $Z_t = z_t$, $Z2 = z_2, \ldots, Z_{K-1} = z_{K-1}$ n .

Sequential simulation is used for the generation of conditional realisations of either a multi-Gaussian random function or any non-Gaussian random function as long as its conditional distribution can be derived. The type of sequential simulation depends upon the way in which the local conditional probability distribution is estimated. For instance, multi-Gaussian kriging yields an estimate of the local conditional probability distribution (lepd) by assuming a normal distribution and estimating the mean and standard deviation. If multi-Gaussian kriging is used in simulation, the algorithm is known as sequential Gaussian simulation. If indicator kriging is used to estimate the lcpd, the algorithm is known as sequential indicator simulation.

2.2 *Sequential Gaussian Simulation*

Parametric non-linear geostatistical methods require the conditional probability distribution of the random variable, which, in practice, is impossible to obtain. For a Gaussian random function with known mean, the conditional distribution of *Z-,* is Gaussian, with mean /,- and variance o £y, where Z_{γ}- is the simple kriging estimator of Z_{γ}- and G ^; is the associated kriging variance. The multi-Gaussian model overcomes this problem by using the normal scores transform of the grades:

$$Z(x) = \phi(Y(x)) \tag{4}$$

$$Y(x) = \phi^{-1}(Z(x)) \quad x \in Orebody \tag{5}$$

One of the most important advantages of multi-Gaussian kriging is that there is no restriction on the type of grade distribution, provided İt is first transformed to a normal distribution. Prior to the simulation, exploratory data analysis helps to reveal whether lack of stationarity, the presence of outliers, clustering of data or spiking exists. Provided that the random function *Y(x)* is also multivariate normal and strictly stationary, sequential Gaussian simulation can be implemented. Otherwise, other procedures should be considered. The hypothesis of multinormality requires each random variable *(Y(x), x* e *Orebody)* to be normally distributed (Olea, 1999). To put it differently, multivariate normality among all variables at all possible spatial locations is assumed. Not only is normality of the one-point conditional distribution function (cdf) required, but also the random function should be multivariate Gaussian. If transformed data are not multivariate normal, the simulated data will not reproduce the characteristics of the original data. Ideally, a transform which is Gaussian all two-point cdfs should be fulfilled. Such a transform is very difficult. A simpler, and more readily verifiable, assumption is bivariate normality. This can be checked from bivariate histograms of sample pairs for a fixed lag during variogram calculation.

The inverse, or back, transformation is performed by linear interpolation. Extreme simulated normal values that lie outside the range of the conditioning data are interpolated using two pre-specified extreme transform pairs. The range is set to the minimum and maximum constraints on the original data values or to narrower values if these constraints are unlikely to be exceeded. The main advantages and disadvantages are as follows (Dowd, 1992):

Advantages:

1. Sequential Gaussian simulation guarantees that data are honoured at their locations because kriging is an exact interpolator and, therefore, yields a zero kriging variance when a datum is estimated. The simulated value is thus drawn from a normal distribution with zero variance and a mean equal to the datum itself. As the conditioning is an integral part of the simulation, no additional step İs necessary.
2. Anisotropics can be handled automatically as part of the kriging process. Kriging with an anisotropic semi-variogram ensures that the anisotropics are imparted to the kriged values, which, in rum are imparted to the simulated values drawn from distributions with means equal to the kriged values.
3. Any covariance function can be implemented.

Disadvantages:

1. The main drawback rests on the assumption of using the intermediary Gaussian distribution. In practice, it is impossible to guarantee a multivariate normal distribution.
2. There is some evidence to suggest that sequential Gaussian simulation produces less variation between successive simulations than other methods such as turning bands.

The selection of application parameters, such as the use of ordinary kriging versus simple kriging, the maximum number of simulated nodes retained for kriging, octant search parameters, and upper and lower extrapolation values can profoundly affect the procedure. Non-stationarity can be taken into account by using kriging with a trend model instead of simple kriging. Chiles & Delfiner (1999) reported that a Gaussian stochastic process with an exponential covariance model is a special case where the method could be applied easily without any approximation for any set of data points or simulated points, if the mean is known.

3 CASE STUDY

Sequential Gaussian simulation is implemented for which 616 data values are available from 27 drillholes and simulated values at 10000 locations

are required. The data are CaO contents taken from the quarry of a cement plant The performance of the method is assessed by its ability to reproduce the specified model parameters and statistics. A stochastic realisation should reproduce the declustered sample histogram and the semi-variogram model, and should coincide with data values at their locations {Figure 2 for semi-variograms and Figure 3 for histograms).

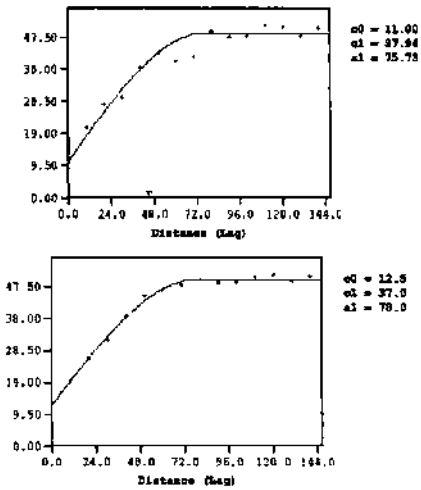

Figure 2. Semi-variograms of dala (above) and simulated field (below).

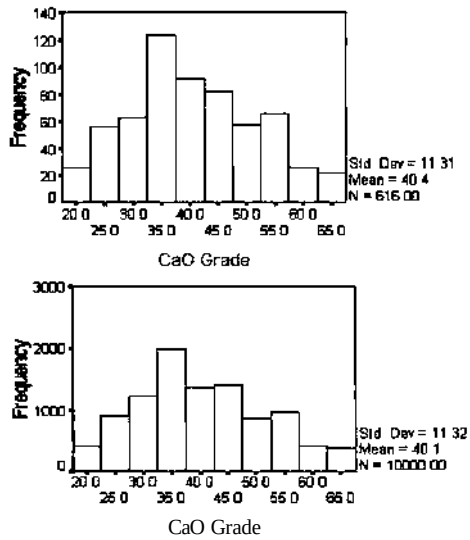

Figure 3. Histograms of data (above) and simulated field (below).

In practice, exact reproduction is impossible because:

- Model statistics are inferred from sample information, which is assumed to represent population parameters. As more data are used in the conditional realisation, the realisation statistics become increasingly similar and closer to the desired statistics.
- Data are subject to measurement error and, thus, measured values are not necessarily the same as the real, in-situ values. Furthermore, the part of the nugget effect that arises from these errors is not reproduced by the realisation semi-variogram.
- When the semi-variogram range is larger than the size of the simulation area, ergodic fluctuations of the semi-variograms of the realisations are especially important. Searching is limited to a maximum of the eight closest original data and the eight closest previously simulated values so as to reduce the computation time and to avoid round-off errors in the kriging matrix. But theoretically, the search should be extended to the semi-variogram range.

Discrepancies between the semi-variograms of the data and those of the simulated values may also arise when:

a. there are no data or previously simulated values for kriging,
b. there are insufficient data and previously simulated values for kriging,
c. the kriging matrix is singular.

4 CONCLUSIONS

As has been seen from the case study, 10 000 locations have been simulated. Histogram and semi-variogram parameters have been fairly reproduced. Selective mining units and planning blocks are obtained by averaging the values of these supports over me larger volumes. The averaging procedure is effectively a change of support operation. The set of blocks obtained by this process constitutes a 3-D block model of me ore body. Simulated values on planning blocks are ready to be submitted to the production scheduler. Note that geostatistical simulation generates only one possible realisation of block grades. Given that the fundamental technical parameters are the ore reserve block grades and tonnages, geostatisical simulation allows the assessment of technical risk arising from these parameters. The simulation procedure is repeated many times. Each simulation produces different block grades and tonnages and thus a different pit shape and size that have a significant effect on the risk analysis.

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