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THE USE OF STEADY-STATE SIMULATORS TO APPROXIMATE DYNAMIC OPERATIONS TN COMMINUTION SYSTEMS

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ABSTRACT: This paper discusses the opportunity of using steady-state simulators for approximating dynamic operations in a comminution system. The particular case of continuous open-circuit ball mill is used to demonstrate the applicability of the approach proposed. The study is divided into two parts. At the first stage, the simulation of an open-circuit ball mill operating without adding water (dry grinding) is attempted, and its dynamic behaviour during a step increase of the ore throughput is successively imitated by two particularly designed simulators (dynamic and steady-state). The response of 80%-passing size in the mill discharge has shown that a good correspondence is achieved between the results obtained at discrete times by the dynamic simulator and those approximated using a sequence of small-scale simulations at steady-state. In the second part, the investigation is extended by three case studies dealing with approximations of dynamic operations in a wet open-mode grinding circuit by use of a commercially available steady-state simulator - MicroSim. Single and simultaneous step changes of ore and water input flowrates are considered at that stage, as well as variations of solids and water holdups in the mill, i.e., the slurry density. The relative influence of each manipulated variable on the mill discharge size distribution is quantified, and realistic responses of 80%-passing class are yielded in all sample problems.

1. INTRODUCTION

The aim of studying dynamic operations in a specific technological system is generally to gain a better understanding of the system, with the view to control it in order to achieve the desired values of certain important output variables, and to satisfy required specifications. To describe and define the detailed system behaviour, engineers must have a firm grasp of process dynamics, as well as the methods available for analyzing their influence on the systems performance. Moreover, insight into the transport phenomena taking place in an industrial system being subjected to various input changes constitutes a valuable tool in surmounting difficulties invariably arising from complicated practical situations.

In spite of major advances in the modelling of ball mill operations, the dynamics of this process are still not well understood by operating staff. Although numerous software packages have recently become commercially available, most of these programs are designed for simulating size reduction processes at steady-state only (Morrison & Moloney, 1992, Espig et al., 1991, Austin et al., 1990, Stange et al., 1988). Some dynamic simulators have also been developed (Herbst et al., 1988, Adel & Sastry, 1988, Bascur & Herbst, 1986), but probably because of the absence of a user-friendly interface or for other unknown reasons, they have found no widespread practical application. The majority of these programs is normally run on main frame computers requiring special skills from process engineers. Likewise, as the dynamic simulators reported in the literature are inaccessible on the market, their prompt implementation on-site (if necessary) is not facile, and their use is mainly directed for educational and limited research purposes. Though the concept of using steady-state simulators to approximate the dynamic behaviour of a grinding system (if accumulation in the unit does not change much) is not new (Austin, 1992), nothing has been published in this regard yet.

It is the objective of this paper to suggest a generalized methodology for approximating dynamic operations in a comminution system by use of steady-state simulators. The particular case of continuous open-circuit ball mill is used to demonstrate the applicability of the approach proposed. However, this study can identify areas for further development work on the dynamic simulation of other mineral processing units, such as crushers, gravity separators, flotation cells, etc

The study was divided into two parts. At the first stage, the simulation of an open-circuit ball mill operating without adding water was attempted, and its dynamic behaviour during a step increase of the ore throughput was successively imitated by two particularly designed simulators (dynamic and steady-state). In the second part, the investigation was extended by three illustrative examples dealing with approximations of dynamic operations in a wet open-mode grinding circuit by use of a commercially available steady-state simulator - MicroSim (Stange et al., 1988). Single and simultaneous step changes of ore and water input flowrates were considered at that stage, as well as variations of solids and water holdups in the mill, i.e., the slurry density.

2. MATHEMATICAL FORMULATION

The grinding process in a tumbling ball mill is complex due to the influence of various factors and forces occurring during the operation of the system, such as the physical and physicochemical properties of the assembly of particles undergoing size reduction, mill conditions (ball load, mill rotational speed, specific power input to the mill), etc.

Let us consider an overflow ball mill operated continuously in an open-circuit mode. The mill is fed an arbitrary feed-size distribution, with a throughput of solids, F. In the case of wet grinding, water is added at a constant flowrate (W) to maintain a given percentage of solids by weight. The water and solids mass flowrates in the mill discharge slurry are WD and FD, respectively. A constant ball load is used, and the rotational speed of the mill is maintained at a certain percentage of the critical speed The water and powder holdups of the mill are indicated by HW and H, respectively. Assume that the material in the ball charge is well mixed, no size classification takes place at the mill exit, and that the kinetics of breakage are linear Since the ore properties are usually characterized on a size discrete basis, a population balance model (Kinneberg and Herbst, 1984) formulated as a system of ordinary differential equations in the time-continuous size-discrete domain was found useful for the purposes of the present study. According to this model, the mass balance on the rth size interval within the mill is given by means of:

$$\frac{d(Hm_i)}{dt} = -S_iHm_i + \sum_{j=1}^{i-1} b_{ij}S_jHm_j + Fm^f_i - FDm^{fd}_i \quad (1)$$

where m, = mass fraction of mill holdup in the rth size class; m^f_j and m^{fd}_j = mass fractions of material in the rth size interval in the mill feed and mill discharge, respectively; Sj = size-discretized selection function giving the fractional rate of breakage out of the rth interval, and bij is the size-discretized breakage function which gives the fraction by weight of daughter fragments from a primary breakage event in the_/th interval that reports to the rth interval.

Since the mill is well mixed, it can be assumed that m^{fd}_i = m;. Rearranging equation (1), and letting FD = (H/ V).Q and ft = H.m;, yields:

$$\frac{dy_i}{dt} = -\left(\frac{Q}{V} + S_i\right)y_i + \sum_{j=1}^{i-1} b_{ij}S_jy_j + Fm^f_i \quad (2)$$

where Q = volumetric flowrate of solids discharging from the mill, and V = volume of powder holdup.

The mass of solids charge in the mill can be computed via

$$H = \sum_{i=1}^{n} y_i \quad (3)$$

As indicated, equation (2) is only valid for a perfectly mixed mill, but it can be applied sequentially to a series of perfect mixers when such a residence time distribution (RTD) is required (Kinneberg and Herbst, 1984) Once the solids holdup is known, the size distribution of the mill discharge at any time *t* can be calculated by use of equation (2) for each size fraction in the particulate assembly, starting at the top size class (/ = 1). However, equation (2) has to be solved numerically. To this end various techniques for numerical integration can be used, such as the Euler/modified Euler method, Heun's method or the Runge-Kutta fourth-order method (Burden et al, 1978).

At steady-state, F = FD, and considering that m; = m^{fd},, equation (1) can be written as follows

$$HS_im^{fd}_i = Fm^f_i - Fm^{fd}_i + H\sum_{j=1}^{i-1} b_{ij}S_jm^{fd}_j \quad (4)$$

Equation (4) represents a more general form of solution of the population balance developed from the conservation condition, that is, accumulation = = input - output + net generation (Herbst, 1979), and describes steady-state operations in a continuous pen-circuit ball mill

In the case when a single step change of water (W^{06}*) or dry solids (Fnew) input flowrate is applied using an impulse manner at time / = 0 (see Fig. 1), the respective transitional processes taking place in the ball mill discharge could be described by the following time-dependent equations:

$$T_1 \frac{dWD^{new}(t)}{dt} + WD^{new}(t) = W^{new}(t - \tau_1) \quad (5)$$

$$T_2 \frac{dFD^{new}(t)}{dt} + FD^{new}(t) = F^{new}(t - \tau_2) \quad (6)$$

where Xj, T_2 = l i m e delay constants experimentally determined for water and solids, respectively; Tj and T2 = time constants allowing for the velocity of transitional processes in the mill, that is:
1"! = L / vi; T_2 = L / v_2; where L = V_m / S is the mill length; V_m = mill volume; S = cross-section area of the mill (S = 2 71 $(D/2)^2$); D = mill diameter; vi = water velocity in the mill; v_2 = powder velocity.

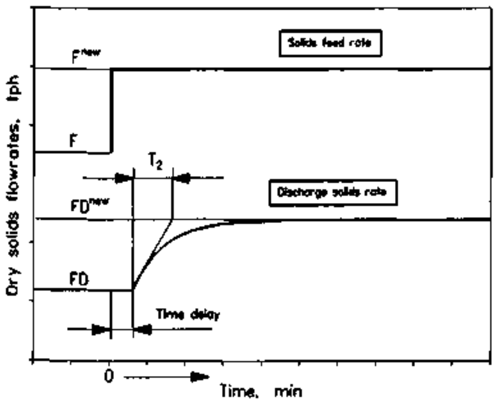

Fig. 1. Step change of dry solids input flowrate applied using an impulse manner at time / = 0, and the respective transitional process taking place in the ball mill discharge.

Equations (5) and (6) are first-order differential equations which can be solved analytically using the Laplace transformation method for initial conditions / > it (k = 1, 2), and in the absence of noise on the system output, the respective auto-correlation functions of water and solids discharge flowrates obtained by that means are as follows[1]

$$WD^{new}(t) = W^{new} - (W^{new} - W) \exp[-(t - \tau_1) / T_1]$$
$$\text{for } t \ge \tau_1 \quad (7)$$

$$FD^{new}(t) = F^{new} - (F^{new} - F) \exp[-(t - \tau_2) / T_2]$$
$$\text{for } t \ge \tau_2 \quad (8)$$

The variances in the water and solids holdups of the mill can be described by the following mass balance equations.

$$\frac{dHW}{dt} = W^{new} - WD^{new} \quad (9)$$

$$\frac{dH}{dt} = \alpha_{2F}F^{new} - \alpha_{2E}FD^{new} \quad (10)$$

where a_2j;, a_2£ = solids fraction by volume in the mill feed and mill exit, respectively; a_{2F} = *PiflPi',* ^{a}2E = *P2d/Pi (*^{ai} steady-state $a_2$pwa$_2$g = const); *Pl* = specific gravity of solid, *pji* = powder density allowing for the bed porosity in the mill feed, *pι&* - the same as above, but related to the ore in mill discharge

After a substitution of equations (7) and (8) into equations (9) and (10), respectively, the analytical solutions of the differential equations (subject to given side conditions) are as follows:

$$HW(t) = HW + (W^{new} - W)\{\theta_1 + \Delta_1 [1 - \exp(-(t - \theta_1) / \Delta_1)]\}$$
$$(11)$$

$$H(t) = H + (F^{new} - F)\{\theta_2 + \Delta_2 [1 - \exp(-(t - \theta_2) / \Delta_2)]\}$$
$$(12)$$

As shown in equations (11) and (12), the time parameters θ_1, θ_2 and Δ_1, Δ_2 might differ from Ti, x_2 and Ti, T_2, which allows for the fact (demonstrated experimentally by Kinneberg and Herbst, 1984 - see Fig. 2) that beyond a certain limit of the feed rates of water and solids, their respective holdups remain constants, i e., if there is no change in the accumulation of the unit, θ_1, θ_2 and Ai, A_2 approach zero In addition, since the interactions between the two phases (water and dry solids) in the mill are not included in equations (9) & (10), A_2 & 6_2 are constants in those cases only when W = const, and similarly Aj & 9j could be constants only when the ore throughput (F) is kept unchanged

To allow for the impact of interactions between the two phases in the mill when step changes of water and solids input flowrates are simultaneously implemented, some corrections of the initial conditions, i e., HW and H, should be included

A general notation for this case has been proposed by the following (Kafarov et al, 1985):

$$HW = f(F^{new}) = HW + c_1 (F^{new} - F) \qquad (13)$$

$$H = f(W^{new}) = H + c_2 (W^{new} - W) \qquad (14)$$

where Ci and c_2 = correction coefficients.

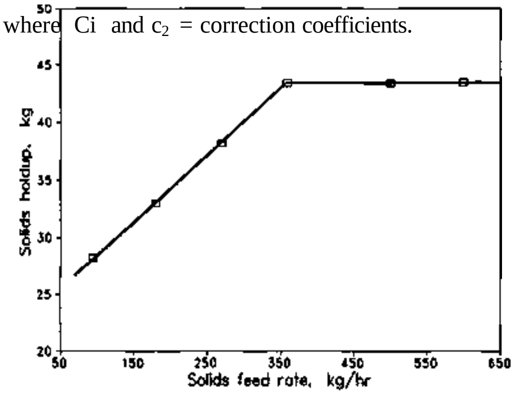

Fig. 2. Solids holdup as a function of solids feed rate.

Substitution of equations (13) and (14) into equations (11) and (12), respectively, provides the following relationships:

$$HW(t) = HW + c_1 (F^{new} - F) + (W^{new} - W) \times \{\theta_1 + \Delta_1 [1 - \exp(-(t - \theta_1)/\Delta_1)]\} \qquad (15)$$

$$H(t) = H + c_2 (W^{new} - W) + (F^{new} - F) \times \{\theta_2 + \Delta_2 [1 - \exp(-(t - \theta_2)/\Delta_2)]\} \qquad (16)$$

Knowing the values of water and solids holdup, as well as the specific gravity of powder *(pi)*, the pulp density in the ball mill, *pp(t)*, can be obtained by the following simple formula (note that pj = 1):

$$\rho_p(t) = \frac{HW(t) + H(t)}{HW(t)/\rho_1 + H(t)/\rho_2} \qquad (17)$$

The respective mean residence times of water, *w, mean (0. and solids, f, mean ('). in the mill during the transitional conditions can be calculated via:

$$t_{W,mean}(t) = HW(t) / WD^{new}(t) \qquad (18)$$

$$t_{S,mean}(t) = H(t) / FD^{new}(t) \qquad (19)$$

3. SAMPLE PROBLEMS

3.1. Case study - *Step change of ore throughput in an open-mode dry grinding system*

Since equations (2) and (4) do not allow for the influence of slurry density on the mill discharge size distribution, an open-circuit ball mill operating without adding water (dry grinding) was considered at that stage. While the input data presented here were hypothetical, they were somewhat similar to those cited in the literature (Stange et al., 1988; Kafarov et al., 1985; Austin et al., 1984):

- **mill geometry: D x L = 1.5 m x 3.0 m;**
- **mill rotational speed = 70 % of critical;**
- **a constant ball load corresponding to about 45 percent dimensionless ball filling is used.**

Assume that at time *t* = 0, the grinding process is set in motion, and the mill is fed an initial solids flowrate of 18 tph with an arbitrary feed-size distribution (Stange et al., 1988) - see Table 1. The feed size 80% passing equals 454 microns, and the average solid density *(pi)* is 2.91 t/m^3.

Table 1: Feed size distribution

Size class No(/)	Size interval (*) (microns)	Size density **(%)**	Weight % less than the **ith** size
1	1700 - 2360	2.40	100.0
2	1180-1700	3.10	97.60
3	850-1180	4.00	94.50
4	600-850	5.00	90.50
5	425-600	6.60	85.50
6	300 - 425	9.10	78.90
7	212-300	13.10	69.80
8	150-212	16.40	56.70
9	106 - 150	12.70	40.30
10	75-106	7.60	27.60
11	53-75	4.90	20.00
12	38-53	3.70	15.10
13	27-38	2.80	11.40
14	19-27	1.80	8.60
15	0-19	6.80	6.80

(#) The size intervals are in geometric s&ccu series, and hereafter the size is denoted by the top size of the interval, i.e., Di = 2360 urn, D_2 = 1700 urn, etc.

It is assumed that the adjustable breakage parameters of the ore being ground - a, ß, *y*, etc. (Austin, 1977), had been experimentally determined in advance from a narrow size fraction feed experiment or tracer tests (Herbst et al., 1988). In this study the size-discretized selection function distribution was modelled by:

$$S_i = S_1 (D_i / D_1^{\alpha}) \qquad (20)$$

where S_1 « selection function for the largest size.

In equation (20) Sj denotes the selection function giving the fractional rate of breakage out of the rth interval, and D, is the top size of the rth particle class.

The cumulative breakage function (for $i > j$) was then modelled by (Stange et al., 1988; Austin, 1977):

$$B_{ij} = \Phi_j \left(\frac{D_{i-1}}{D_j}\right)^{\gamma} + (1 - \Phi_j)\left(\frac{D_{i-1}}{D_j}\right)^{\beta} \qquad (21)$$

where Oj = function of theyth size being broken and the topmost size fraction (class 1):

$$\Phi_j = \Phi_1 \left(\frac{D_j}{D_1}\right)^{-\delta} \qquad (\text{for } j = 2, 15) \qquad (22)$$

It was also assumed that the size-discretized selection and breakage functions were insensitive to the mill conditions, as well as independent of mill content size distribution and time, hence, 8 = 0. By use of equations (21) and (22), considering that the size intervals are in geometric screen series, that is, Dj = Di R G - 1), $R = 1 / 2^{1/s}$, it follows that the breakage functions can be dimensionally normalized:

$$B_{ij} = B_{i+1,j+1} = B_{i+2,j+2} = B_{i+3,j+3} = ..., \text{etc.} \qquad (23)$$

The size-discretized breakage functions, bii, which give the fraction by weight of daughter fragments from a primary breakage event in the first interval that reports to the rth interval, were determined by:

$$b_{i1} = B_{i1} - B_{i+1,1} \qquad (\text{for } 14 \geq i \geq 1) \qquad (24)$$

$$b_{15,1} = 1 - \sum_{i=1}^{14} b_{i,1} \qquad (25)$$

(note that $b_{ij} = 0$ when $i = j$)

It is evident from eqs. (23) and (24) that:

$$b_{ij} = b_{i+1,j+1} = b_{i+2,j+2} = ..., \text{etc.} \qquad (26)$$

During the computational procedure described above, no scale-up relationships were used, nor yet any residence time distribution, as well as the liberation phenomenon was not modelled.

Thereafter, two computer programs were developed. Using a recursive procedure by solving equation (2) for each size fraction in the assembly of particles, starting at the top class, dynamic operations in a grinding circuit can be simulated through the use of the first program (DYNSIM), while the second one (STATSIM) represents a steady-state simulator utilizing equation (4) within its computational framework. The software was coded in Microsoft QBasic. The technique used in solving the ordinary differential equations for each size interval was the fourth-order Runge-Kutta method. To ensure a correct mass balance at any time *t,* i.e., to meet the specification that the sum of all the classes totals 100 percent, an optimization search similar to the method of dichotomy was included in the algorithm of the dynamic simulator. Some other techniques, such as the Fibonaccian optimal search, interpolation algorithms, etc., could also be applied, and thus, a faster convergence of the solution to be achieved. A subroutine for calculating the set of size-discretized selection function distribution and breakage distribution matrixes (at given input values of adjustable size reduction parameters of the ore being ground, namely, a, ß, y, etc.) was included in both software programmes. Furthermore, for simplicity of computations, a constant powder holdup of one metric ton was adopted.

Considering the mean residence time of solids at steady-state ($t_{s,mean}$ = 3.33 min), it was assumed that the mill had started to discharge material three minutes after putting the system into operation By consecutive runs of the dynamic simulator using an integration step of 0.25 min, numerous size distributions in the mill discharge at given regular times were simulated, and a part of these time-discretized distributions is portrayed in Figure 3.

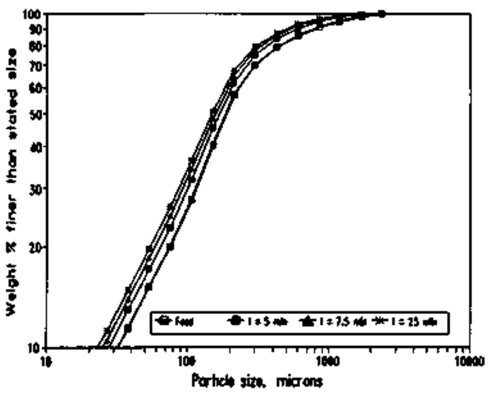

Fig. 3. Size distributions in the mill discharge at given times till attainment of steady-state.

Whereas the choice of a suitable integration interval is discussed in detail by (Sanchez & Amauta, 1989), it must be emphasized here that if a large time increment was chosen, then quite erroneous results could be obtained since the error was significant, and some oscillations occurred.

The equilibrium for all size fractions in the assembly, of particles, i.e., the steady-state of the grinding circuit, was reached at about the 35th minute after setting the system in motion, and the respective mill discharge size distribution was identical to that one obtained by the STATSIM simulator.

It was further assumed that at time t = 35 min, a step increase of the ore feed rate of 6 tph, i.e., pnew - 24 tph, was introduced using an impulse manner. As indicated, no changes of the accumulation in the unit were considered. The resulting output response of the system at discrete time intervals was simulated by equation (6), and the respective auto-correlation function of the discharging powder flowrate is depicted in Figure 4.

While the time delay constant was arbitrarily selected fa = 185 min), the T2 value was chosen to equal the mean residence time of the solids at the new steady state, that is: T_2 =2.5 min. In fact, the choice of T 2 was important and conditioned by the prerequisite that the period for attainment of equilibrium of the total mass flowrate at the mill exit (Fig. 4) had to be adequate (corresponding) to that time interval for which the flowsheet again comes to steady-state after the throughput increase.

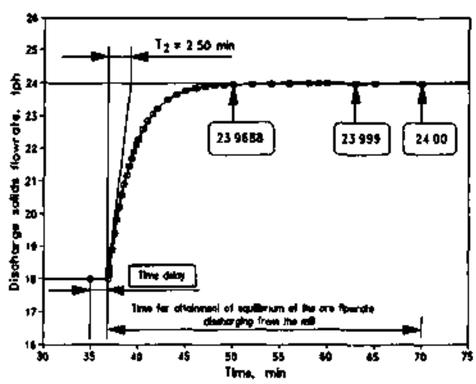

Fig. 4. Auto-correlation function of the discharging powder flowrate.

The dynamic behaviour of the milling circuit after applying the step change, was simulated by the program DYNSIM. To this end, starting from / = 36 85 min, the simulator was consecutively run within discrete time intervals [4, 4+1], with a time increment öt = »i+i - «V During each rth run (i = 1, 35), the arithmetic average of the rth and $(i$ + l)th values of the output solids flowrate, calculated via equation (6), was used.

On the other hand, the STATSIM simulator was successively run with ore throughputs equaling the respective discharging solids rates computed by equation (6), and thus, a set of different size distributions of the mill product (at steady-state) was determined. Figure 5 portrays a comparison between the DYNSIM simulator response of 80%-passing size emerged due to the ore throughput increase, and the same response approximated by the program STATSIM by use of a sequence of small-scale simulations at steady-state.

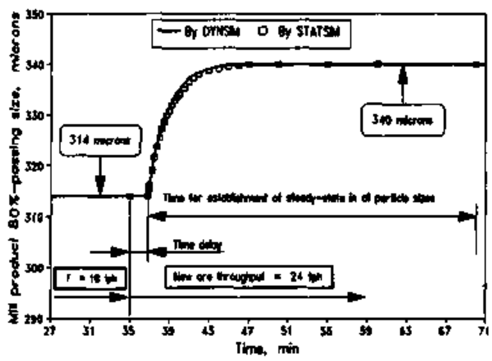

Fig. 5. Comparison between the response of 80%-passing size simulated by the DYNSIM simulator, and the same response approximated by the program STATSIM.

It can be seen from Figure 5 that a very good correspondence is achieved between the results obtained by the dynamic simulator DYNSIM utilizing equation (2), and those predicted through the consecutive use of the steady-state simulator (STATSIM). The time for establishment of steady-state in all particle sizes is also shown in Fig. 5, and it turned out to be almost the same as the duration needed for the discharging ore flowrate to attain its equilibrium (cf. Fig. 4 & Fig. 5). A comparison of relationships ascertained by both simulators between the size 80% passing in the milling product and the output mass solids rate is given in Fig. 6, while the sensitivity of the response of this particular size to the selection of the time constant allowing for the velocity of transitional processes in the unit (T_2) is shown in Figure 7. Moreover, the equilibrium size

distribution at the mill exit obtained by the program using the Runge-Kutta integration algorithm, was identical to that determined by the simulator STATSIM for F = FD = 24 tph.

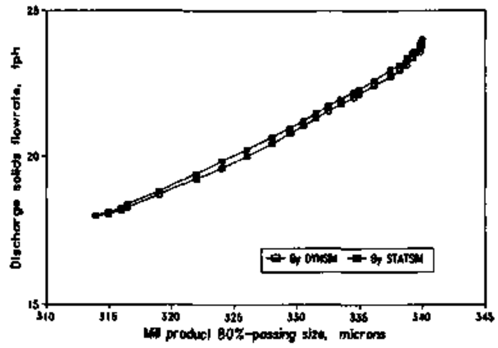

Fig. 6. Comparison of relationships obtained by the two simulators between the discharge solids flowrate and 80%-passing size in the mill product.

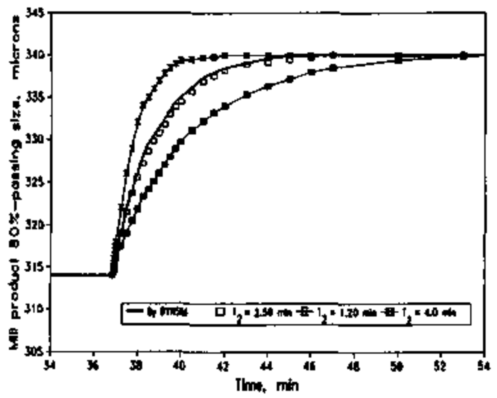

Fig. 7. Sensitivity of the response of 80%-passing size to the selection of the time constant T_2.

Approximations of dynamic operations in an open-mode wet milling circuit by use of a commercially available steady-state simulator - MicroSim (Stange et al., 1988; Stange, 1989) are illustrated in the following three case studies. MicroSim is a powerful but user-friendly software package for simulating various minerals processing operations. It is designed specifically for MS-DOS compatible microcomputers and contains models for the simulation of most ore-dressing and coal preparation processes. The package allows the user interactive drawing of virtually unlimited unit configurations including rod and ball mills, hydrocyclones, screens, etc. MicroSim is destined to engineers who are familiar with plant **Operations but not necessarily skilled in modelling and computing.**

The models used in MicroSim for simulating a ball mill are based on the breakage-selection function approach (Austin, 1977). No scale-up relationships are provided, and the liberation phenomenon is not modelled. The solids holdup, i.e., the mean residence time of powder in the mill, may be selected freely by the user, and the mill geometries need not to be specified.

Single and simultaneous step changes of ore and water input flowrates were considered at this stage, as well as variations of solids and water holdups in the mill, i.e., the slurry density. In the cases described below it was assumed that the size reduction parameters of the ore being ground were insensitive to the mill conditions, as well as independent of size content within the mill and time. However, the investigation could be extended by treating grinding systems in which the size reduction parameters, particularly selection functions, are dependent on mill rotational speed (Austin et al., 1984), solids holdup in the unit (Herbst & Fuerstenau, 1973), time, etc. Other software packages (Austin et al., 1990) could be attempted as well.

The input data being invariable during the computational procedures in all the following sample problems were:

Average solid density: P2 = 2.91 t/m^3;
Values of breakage parameters of the ore being ground:
a = 0.91 min^{-1}; $\beta = 2.95$; $\gamma = 0.61$; $\delta = 0.0$;
Φ_1 (of the largest size) = 0.63;
Selection function at the top size, Sj = 2.35 min^{-1}.

3.2. Case study - ***Step change of ore throughput in an open-mode wet grinding system***

Consider a tumbling ball mill operated continuously in an open-circuit mode. The mill is fed an ore with the feed-size distribution as shown in Table 2, with a 12 tph throughput of solids, i.e., F = 12 tph.

Water is added at a constant flowrate (W = 10 tph) to maintain a 54.55 % mass percentage solids Assume that at some time, water and solids holdups were measured directly (by stopping and emptying the slurry in the mill), and that their respective values were as follows:

Water holdup in the mill: HW = 0.751;
Solids holdup in the mill: H = 0 901

Hence, the mean residence time of solids in the mill (at steady-state) can be calculated by equation (19):

t_s mean = H / F = 0.90 [t] / 0.20 [t/min] = 4.50 min

Table 2 Feed size distribution

Particle size (microns)	Size density (%)	Wt % less than ith size
3350	1 00	100
2360	3 00	99
1700	16 00	96
1180	15 00	80
850	15 00	65
600	7 00	50
425	5 00	43
300	5 00	38
212	4 00	33
150	4 00	29
106	4 00	25
75	7 00	21
53	3 00	14
38	2 20	11
27	2 00	8 8 (#)
19	1 60	6 8 (#)
13	1 20	5 2 (#)
9 50	4 00	4 0 (#)

W Extrapolated (Size 80%-passing in the feed =1180 um)

Assume that at time $t = 0$, a step increase of input powder flowrate of 6 tph ($F^{new} = 18$ tph) is realized, and the mill feed has a new mass percentage solids, which equals 64 29 % Assume also that the time delays and constants which allow for the velocity of respective transitional processes of the output ore flowrate and solids holdup in the mill were measured and indirectly determined (by tracer results) through accurate monitoring at the mill discharge Adopt that the following values of time constants were experimentally ascertained

$$\tau_2 \approx \theta_2 = 1\ 25 \text{ min}, T_2 = 3\ 33 \text{ min, and } \Delta_2 \approx 3 \text{ min}$$

Further, the dynamic behaviours of some system variables, i e, discharging ore rate, solids holdup, pulp density, were estimated by the application of equations (8), (12) and (17), respectively, whilst the response of mill product 80%-passing size to the throughput increase was simulated by MicroSim The results obtained are illustrated in Figures 8 and 9 The former plot shows the dynamic increase of the amount of coarse particles at the ball mill exit along with the transitional change of the pulp density Despite the increase of the accumulation within the unit (Fig 9), with a higher solids flowrate, less size reduction is effected since the material is in the mill for a shorter time (less than 4 5 min)

It is evident that the step change of the throughput has a marked influence on the performance of this circuit - the mill product 80%-passing size increases from 109 to 148 microns Though based on hypothetical ore attributes and model parameters, the simulation produces realistic results that compare well with milling practice

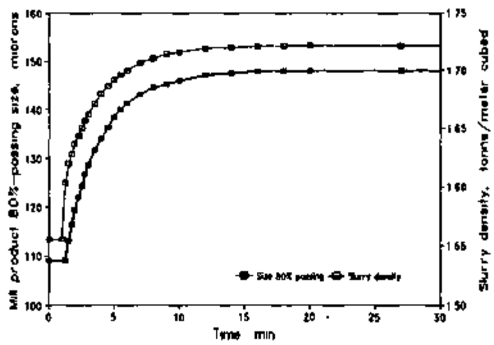

Fig 8 Dynamic behaviour of the 80%-passing size along with the time-dependent change of pulp density after the step increase of input solids flowrate

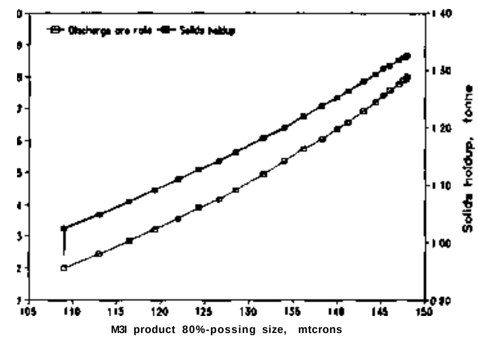

Fig 9 Correlation between 80%-passing size in the mill discharge, and output solids rate & solids holdup, respectively

3 3 Case study - *Step change of water input flowrate in an open-mode wet grinding system*

Assume that at time / = 60 min, i e, when the circuit had already reached its new steady-state after the ore throughput increase, a step augmentation of water addition flowrate of 5 tph (W[1]»[6]* = 15 tph) is applied using an impulse manner, and the mill feed has a mass percent solids, which again equals 54 55 % The values of breakage parameters of the ore being ground, as well as the feed size distribution remain the same as those in the previous example (note that the ore throughput, F = 18 tph, is kept constant in this case) Assume also that the time constants allowing fur the transitional processes of water holdup in the mill $(\theta_1$ & $\Delta_1)$i, and those for the output water rate (TJ and Tj), had been ascertained through accurate monitoring and measurements at the mill discharge (or by means of other methods for system identification), and that the respective values thus extracted were as follows:

$$\theta_1 \approx \tau_1 = 1 \text{ min, and } \Delta_1 \approx T_1 = 3.50 \text{ min.}$$

The dynamic characteristics of system components (discharging water flowrate, water holdup, pulp density) were estimated by the application of equations (7), (11) and (17), respectively. The response of mill product 80%-passing size to the increase of the water addition flowrate was simulated by the steady-state simulator MicroSim. The results obtained in this way are portrayed in Figure 10 and Figure 11.

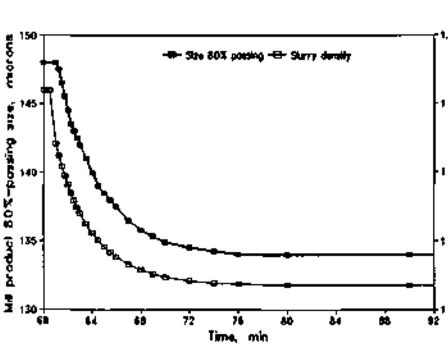

Fig. 10. Dynamic response of the 80%-passing size along with the time-dependent change of pulp density after the step increase of input water flowrate.

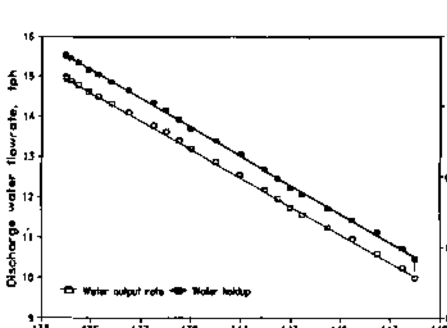

Fig. 11. Correlation between mill product 80%-passing size, and discharge water flowrate & water holdup, respectively.

As expected, there is a certain increase in the circuit performance due to the increase of water input rate since more fine material is produced - the mill product 80%-passing size decreases from 148 to 134 microns - Figure 10. The explanation of this phenomenon is well known - less mass percent solids in a milling circuit is generally considered to be more efficient because the fine particles are suspended in the liquid and are effectively removed from the grinding zone. Moreover, the water allows better transfer of the mechanical action of the tumbling balls to the stressing of the particles (Austin et al, 1984). However, wet grinding with a low ratio of solid to water (slurry density) is ineffective from an economical point of view. On the other hand, when the pulp density is much high, the mill charge becomes thick and viscous. Hence, there is an optimum liquid-to-solids ratio giving a maximum production rate under an optimal filling level of the material within the mill.

3.4. Case study - *Simultaneous step changes of input ore and water flowrates m an open-mode wet grinding system*

It can be supposed that the augmented and heavier loading in the unit, emerged in consequence of the successive step alterations of ore and water feed rates, has resulted, to some extent, in an increase of power consumption (specific energy per ton of ore ground) required to drive the mill. Assume that for this reason, at time t = 120 min, the operator had decided to reinstate the operating conditions in their respective initial settings, that is, F = 12 tph and W = 10 tph. In this case, when the interaction between the two phases (water and solids) in the mill is to be taken into consideration, some corrections of HW and H should be included, cf. eqs. (13) and (14).

Assume that through accurate monitoring and measurements at the mill discharge some experimental data were obtained, and by adjusting those data (eg., in a least-squares sense), the following values of time constants and correction coefficients had been determined

$$\theta_1 \approx \tau_1 = 1 \text{ min, } \Delta_1 \approx T_1 = 3.20 \text{ min; } \theta_2 = 1 \text{ min,}$$
$$\tau_2 = 1.25 \text{ min, } \Delta_2 = 2.75 \text{ min, and } T_2 = 3 \text{ min,}$$
$$c_1 = 0\ 25, \text{ and } c_2 = 0\ 30$$

Further, the dynamic behaviours of system variables, i.e, discharging water and ore rates, water and solids holdups in the mill, pulp density, were modelled by the application of equations (7), (8), (11), (12) and (IT), respectively. The response of mill product 80%-passing size to the simultaneous step decrease of water and solids feed rates was simulated by MicroSim, moreover shown in Figure 12 It must be noted that due to the different time delay constants for water and solids discharge flowrates $(\tau_1 \neq \tau_2)$, there is a small peak in the trend of this particular class during the initial transitional period - see Fig 12 If 01 differed θ_2, e g , $\theta_2 > \theta_1$, then similar peak in the dynamic response of pulp density could be observed in the opening stage

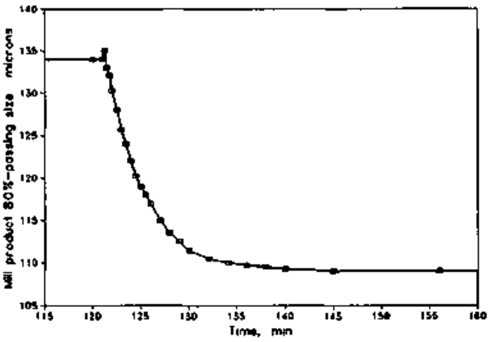

Fig 12 Dynamic response of mill product 80%-passing size to the simultaneous step decrease of water and solids feed rates

The overall tides of variances in the system components during the process variations are depicted in Figures 13 and 14

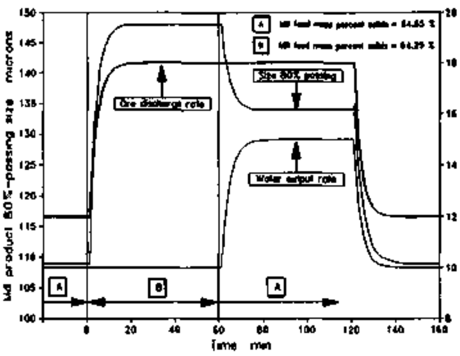

Fig 13 Overall tides of variances in mill product 80%-passing size, ore discharge rate and water output flowrate

From these plots the relative influence of each operating variable on the size distribution of the circuit output can be quantified It can be seen that a change in the water addition rate exerts an effect on the mill product 80%-passing size, considerably less in magnitude than the effect observed from a similar change of the solids throughput - Figures 13 & 14 It must be pointed out that during the simultaneous step decrease of input ore and water flowrates the slurry density varies slightly as the liquid-to-solids ratio in the mill remains almost the same - see Fig 14

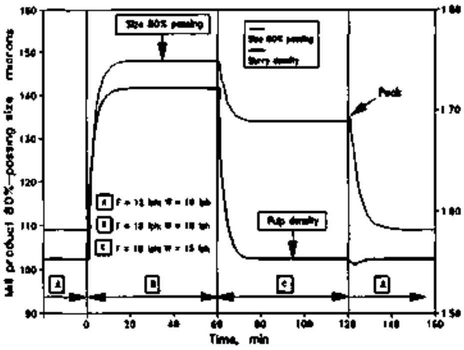

Fig 14 Illustration of the relative influence of each manipulated variable (F & W) on the mill product 80%-passing size and pulp density during their single and simultaneous step changes

It must be pointed out that during the simultaneous step decrease of input ore and water flowrates the slurry density varies slightly as the liquid-to-solids ratio in the mill remains almost the same However, despite this constant dilution, the mill product 80%-passing size decreases from 134 to 109 microns - Figure 14 This fact can be practically explained by the presence of a lower level of material within the mill It is commonly accepted that if the filling level of solids is raised (higher than an optimal limit), the cushioning action is increased, and the production of fines in the milling circuit is reduced Thus, it was shown by all the illustrative examples presented that the simulated results compare favourably with those encountered in industry Therefore, the dynamic behaviour of an open-mode grinding system can be realistically approximated and studied on the basis of models proposed, as well as through a successive use of small-scale simulations at steady-state

4 DISCUSSION

In this paper suitable mathematical expressions relating the input and output of the most important components of a continuous open-circuit ball mill are suggested to represent the dynamic behaviour of the system These models are - derived from a fundamental mathematical description of the process, and in contrast to yet published mass transfer laws relating solids holdup in a mill and throughput (Austin et al, 1984), they are not developed empirically by analyzing of industrial scale data. The models presented in this paper include some parameters the values of which have to be experimentally ascertained, however. The measurement of time constants allowing for the velocity of transitional processes in a mill (9j, TJ, AJ, Tj, j = 1, 2) is meaningful, as the operating conditions may vary greatly. Various methods of system identification applicable to the determination of transfer functions by on-line testing during normal plant operation are available (Schwarzenbach & Gill, 1984; Isermann, 1981). For example, in the absence of noise, the system can be subjected to an input change in the form of a step function, and the resulting response recorded at regular times. The curve thus obtained would enable values for time constants to be readily evaluated. However, most of the experimental procedures for system identification are not always practical for a grinding circuit because of limitations imposed by the existence of random noise due to equipment disturbances, inconsistency of the feed, solids and water loads, environment, etc. In this case, some recursive parameter estimation algorithm (Herbst et al., 1986) that compares the process measurements and model information under a given performance criterion (least-square estimate, maximum likelihood estimate, etc.) can be used for on-line applications. One estimation technique which has been applied successfully in several mineral processing operations (Herbst et al., 1986; Herbst & Oblad, 1986) is the extended Kalman Filter (Kalman and Bucy, 1960) representing an optimal least-squares estimator for linear dynamic systems driven by white noise. A new method for system identification based on the use of neural nets (Aldrich & Van Deventer, 1993) could be promising and worth while as well, especially when the model derived from a mathematical description of the process is of high-order and nonlinear. Various neural net structures can be designed to filter the noise associated with the raw process data, based on spatial and temporal redundancies in those data.

5. SUMMARY

A generalized approach for approximating dynamic operations in a comminution system through a successive use of small-scale simulations at steady-state is presented. The particular case of continuous open-circuit ball milling is used to demonstrate the applicability of the methodology proposed. Single and simultaneous step changes of ore and water input flowrates are considered, as well as variations of solids and water holdups in the mill, i.e., the slurry density. Illustrative examples dealing with approximations of dynamic operations in dry & wet open-mode grinding circuits are given, and realistic results are yielded in all sample problems. Suitable mathematical expressions allowing for the influence of operating conditions on the mill performance are suggested. It is shown that the knowledge stored in equations (5) + (16), describing the principal sub-processes occurring in a continuous open-circuit ball mill, along with the use of a state-state simulator could allow the mineral processing engineer to develop a real representation of the system dynamics. Sensitivity analyses can be conducted, and a variety of alternative strategies can be tried in a very short time (Bascur & Herbst, 1986). The generalized methodology can be used if the accumulation in the unit does not change too much, it would therefore be appropriate for evaluating control strategies in which variations in operating conditions are not dramatic.

Interested readers could either develop their own steady-state simulator using the model equations described here/in the literature, or can acquire a commercially available software package, like MicroSim.

REFERENCES

Adel, G., and Sastry, K. V S., 1988. Software for Dynamic Simulaton of Mineral Processes, in *Proceedings of the 2nd International Symposium on Beneflciation,* Kanpur, India.

Aldrich, C, and Van Deventer, J. S. J., 1993. The Use of Neural Nets to Detect Systematic Errors in Process Systems, *International Journal of Mineral Processing,* vol. 39, 173-197.

Austin, L. G., 1977 *The Design of Grinding Circuits,* Chap. 7., SAIMM.

Austin, L. G, Klimpel, R. R, and Luckie, P. T., 1984. *The Process Engineering of Size Reduction: Ball Milling,* SME-AIME, New York.

Austin, L. G., Yildirim, K., et al., 1990. *Grinding Circuit Simulator for Single and Two Stage Ball Mill Circuits - User Guide,* The Mineral Processing Section, The Mineral Engineering Department, The Pennsylvania State University, USA.

Austin, L. G., 1992. Private Communication.

Bascur, O. A., and Herbst, J A., Dynamic Simulators for Training Personnel in the Control of Grinding/Flotation Systems, in *Proc. of the 1st IFAC Symposium on Automation for Mineral Resource Development,* Norrie, A W and Turner, D. R (eds), Pergamon Press, Oxford, 315-324

Burden, L. R, Faires, J D, and Reynolds, A. C, 1978. *Numerical Analysis,* Prindle, Weber & Schmidt Publ., Boston.

Espig, D., Reinsch, V et al., 1991. Software Solution for the Complex Analysis and Simulation in Mineral Processing, in the *XVIIth IMPC Preprints,* Vol. 1, Bergakademie Freiberg, Germany, 247-257

Herbst, J. A., and Fuerstenau, D. W., 1973. Mathematical Simulation of Dry Ball Milling Using Specific Power Information, *Trans. AIME,* vol. 254, 343-348.

Herbst, J A, 1979. Multiparticle Metallurgical Systems, Sec. 2.1. in *Rate Processes in Extractive Metallurgy,* Sohn, H. Y., and Wadsworth, M E. (eds), Plenum Press, New York, 43-95.

Herbst, J. A., Hales, L B., Zaragoza, R., 1986. Strategies for The Control of Flotation Plants, Chap. 36 in *Installation of Concentration & Dewatering Circuits,* Mular, A. L., & Anderson, M. A. (co-eds), SME, Littleton, Colorado, 548-569.

Herbst, J. A., Oblad, A. E., 1986. Modern Control Theory Applied to Crushing. Part 1 : Development of a Dynamic Model for a Cone Crusher and Optimal Estimation of Crusher Operating Variables, in *Proc. of the 1st IFAC Symposium on Automation for Mineral Resource Development,* Norrie, A. W. and Turner, D. R. (eds), Pergamon Press, Oxford, 301-307.

Herbst, J. A, Alba J., F., Pate, W. T., & Oblad, A E., 1988. Optimal Control of Comminution Operations, *International Journal of Mineral Processing,* vol. 22, No 1-4, 275-296.

Herbst, J. A., et al., 1988 A Hierarchical Procedure for Mass Balance Closure and Parameter Estimation: Application to Ball Mill Grinding, in *Proceedings of the XVI IMPC,* Vol. 2, Forssberg, K. S E. (ed.), Elsevier, Amsterdam, 1719-1732.

Isermann, R., 1981. *Digital Control Systems,* Springer-Verlag, New York

Kafarov, V V. et al., 1985. *System Analysis of Chemical-Technological Processes. Processes of Size Reduction and Mixing of Powder Materials,* Nauka, Moscow, (in Russian).

Kalman, R. E, Bucy, R. S., 1960. New Results in Linear Filtering and Prediction Theory, *Journal of Baste Engineering,* vol. 82, 35-40

Kinneberg, D J, and Herbst, J. A., 1984. A Comparison of Linear and Nonlinear Models for Open-Circuit Ball Mill Grinding, *International Journal of Mineral Processing,* vol. 13, 143-165

Morrison, R. D., Moloney, M. J., 1992 JKMR Software Packages, Paper presented at the *Conference on Computing 'Solutions,* Perth, Western Australia, March 18-20, 1992.

Sanchez, E. M, and Amauta, W., 1989. Solutions to the System of Batch Ball Mill Grinding Equations, in *Proceedings of the International Symposium on Computer Software in Chemical and Extractive Metallurgy,* Montreal, Canada, August 28-31, 1988, Thompson, W T., Ajersch, F., and Eriksson, G. (co-eds.), Pergamon Press, New York, 153-160.

Schwarzenbach, J, and Gill, K. F., 1984. *System Modelling and Control,* Edward Arnold Pubis. New York.

Stange, W., Cilliers, J. J., and King, R. P., 1988. *MicroSim version 3.0 - User Manual,* Department of Metallurgy and Materials Science, University of Witwatersrand, RSA.

Stange, W., 1989. Strategies for More Flexible Sequential - Modular Simulation, Chap. 73 in *21-st APCOM Proceedings,* Weiss, A. (ed.), SME, New York, 811-825.

LIST OF SYMBOLS USED

bij = size-discretized breakage function which gives the fraction by weight of daughter fragments from a primary breakage event in the/th interval that reports to the rth size interval

By = cumulative breakage function distribution which gives the fraction by weight of primary progeny fragment from parent particles in theyth size interval finer than size /

Ci = correction coefficient, cf. equations (13) & (15)

C2 = correction coefficient, cf. equations (14) & (16)

D = mill diameter

Dj = topmost size of the rth size interval

F = ore throughput

pnew = step change of solids throughput (new feed rate of solids)

FD = solids mass flowrate in the mill discharge

prjnew = new solids mass flowrate in the mill discharge (at steady-state) after the step change of ore throughput

FDnew(t) = dynamic response of solids mass flowrate in the mill discharge

H = mass of solids charge in the mill

jjnew = new mass 0f solids charge in the mill (at steady-state) after the step change of ore throughput

Hnew(t) = dynamic response of solids charge in the mill during the transitional process

HW = water holdup in the mill

HWnew = new water holdup in the mill (at steady-state) after the step change of input water rate

$HW^{new}(t)$ = dynamic response of water holdup in the mill during the transitional process
$L = V_m / S$ mill length
m_i = mass fraction of mill holdup in size class /
m^f_i = mass fraction of material in the rth size interval in the mill feed
m^0_i = mass fraction of material in the rth size interval in the mill discharge
Q = volumetric flowrate of solids discharging from the mill
$R = 1/2^{1/2} \approx 0\ 7071$
$S = 2\pi (D/2)^2$ cross-section area of the mill
S, = size-discretized selection function giving the fractional rate of breakage out of the rth size interval
t = time
$t_{S,mean}$ = mean residence time of solids in the mill
$t_{w,mean}$ = mean residence time of water in the mill
$T_1 = L / v_1$ time constant allowing for the velocity of transitional processes of output water flowrate
$T_2 = L / v_2$ time constant allowing for the velocity of transitional processes of output solids flowrate
v_1 = water velocity in the mill
v_2 = powder velocity in the mill
V = volume of solids holdup in the mill
V_m = mill volume
W = input water flowrate
W^{new} = step change of water addition flowrate (new feed rate of water)
WD = water mass flowrate in the mill discharge
WD^{new} = new water mass flowrate in the mill discharge (at steady-state) after the step change of water addition rate
$WD^{new}(t)$ = dynamic response of water mass flowrate in the mill discharge
y, = H mj mass of rth size class in the mill holdup

Greek letters

a = first-order breakage constant
a_{2E} = *Pli I Pi* solids fraction by volume in the mill exit
a_{2F} = *Pit I Pi* solids fraction by volume in the mill feed
ß = adjustable size reduction parameter of the ore being ground
δ = adjustable size reduction parameter of the ore being ground
$\delta_i = t_{i+1} - t_i$ time increment used in numerical integration
Δ_1 = time constant allowing for the velocity of dynamic changes of water holdup in the mill
Δ_2 = time constant allowing for the velocity of dynamic changes of solids charge in the mill
Φ_i = function of the rth size being broken and the top size, cf equation (22)
γ = adjustable size reduction parameter of the ore being ground
$\pi = 3\ 1415927$
θ_1 = time delay constant for water holdup in the mill
θ_2 = time delay constant for solids holdup in the mill
ρ_1 = 1 specific gravity of water
ρ_2 = average specific gravity of solid
ρ_{2d} = powder density allowing for the bed porosity in the mill discharge
Plf = powder density allowing for the bed porosity in the mill feed
ρ_p = pulp density in the ball mill
τ_1 = time delay constant for water mass flowrate discharging from the mill
τ_2 = time delay constant for solids rate discharging from the mill