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Yielding Pillar Concept and its Design

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ABSTRACT: Yielding pillar theory and the comparative performance of yield-pillar-protected roadways to critical and stable pillar systems are described. The applicability of empirical methods, the mine stiffness concept, enhanced confined core concept and numerical modelling in designing a yield pillar during the development stage of mining in two-entry systems was investigated and the drawbacks of each method are outlined. These methods were compared by evaluating published data from field measurements. Finite difference models were arranged for estimation of strata stiffness at the pillar location for comparison with the post-failure slopes of pillars of 5, 7.5 and 10 m İn width, which are the intended preliminary design for a UK coal mine. The estimated pillar width ranges of the methods for this example were compared. The findings are verified by those from previous investigations.

1 INTRODUCTION

The safety and productivity of the longwall mining method depend on the maintainence of ground control in the gate entries. The stability of a development roadway serving a new panel is ensured by leaving a stable pillar of such a width that the stresses induced by the nearby excavation do not significantly influence the level of original virgin vertical stress over the working region of a new panel. As longwall mining progresses to greater depths, conventional stable pillar designs require greater and greater pillar width. Widths in excess of 60 m are necessary to provide ground control under 600 m depth of cover. The narrow pillar, designed to yield during longwall mining operations, has been popular especially in the US and recently in the UK. It is used to increase productivity and to minimise problems associated with mining at greater depths. The pillar between the gate entries is designed such that the width/height ratio provides a destressed zone over the supply gate of the new panel by gradual yielding and transfer of the potentially dangerous stress concentrations to the adjacent pillar.

This work arose from the necessity of leaving a pillar between the gate entries of extraction and development panels in Bilsthorpe coal mine. This was the case after an extensive fall of the supply gate of a development panel driven immediately adjacent to the loader gate of an excavation panel had occurred. First, the theoretical basis for yielding pillars is explained. Then, a preliminary design for such a pillar with a view to safer supply gate location (Figure 1) is sought by evaluating various design methods and previous measurements of different field site applications. A finite-difference-method-based two-dimensional code, FLAC, was also used to determine the local mine stiffness by applying force to the roof for various pillar width configurations. Numerical analysis was also performed for the same configurations in order to investigate the magnitude of vertical stresses over the pillar after yielding.

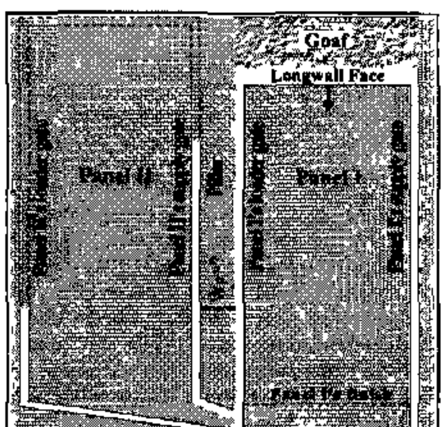

Figure 1. The layout of workings in Bilsthorpe colliery for preliminary design of a yield pillar

2 YIELDING PILLAR THEORY

The yielding pillar concept was first developed for room and pillar mining to create a destressed zone in the working area. The size of the pillars is determined so that yielding of the pillars is ensured and loads are transferred from the working area to the adjacent barrier pillars. The yield pillar system has proven more stable and more economic than abutment pillars at mining depths greater than 450 m. However, there have been documented cases of yield pillar successes at mining depths of less then 450 m (Kripakov et al., 1994). A conceptualised relationship between yield, critical and stable pillars in terms of gate road performance is illustrated in Figure 2. The horizontal axis represents the minimum performance standard differentiating stable gate road configurations from unstable configurations. A pillar design whose performance falls above the horizontal axis is considered successful, while a design whose performance falls below the horizontal axis is considered unsuccessful. The deterioration of ground conditions is generally more gradual for abutment pillars as pillar size decreases. Changes in performance are witnessed by the onset of minor floor heave, an increase in audible coal popping and an increase in the frequency of roof-related problems (Koehler et al., 1996).

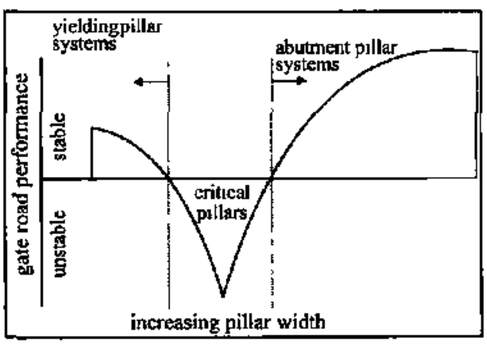

Figure 2. Conceptualisation of the yield pillar concept

Incorrect sizing of yield pillars can result in worsened entry conditions. Mark (1990) stated that between yield and abutment pillar sizes, there are intermediate pillar sizes that are too stiff to yield but also too small to redistribute stresses effectively within themselves. Such pillars, called "critical pillars", maximise disturbance of the surrounding ground. Field observation and stress measurements in a US coal mine showed that a 16.8-m pillar width at a depth of 800 m is a critical pillar design. Pillar sizes of 12.2 m and 10.6 m were found to be yielding pillars (Koehler et al., 1996). Studies İn British coal mines have shown that narrow pillars with widths in the range of 10-30 m result in a high degree of gate roadway closure, producing a 70% change in cross-sectional area (Whittaker & Singh, 1979). Figure 3 shows that when very large conventional pillars were used, better conditions were common. However, yield pillars exhibit better performance than critical-size pillars.

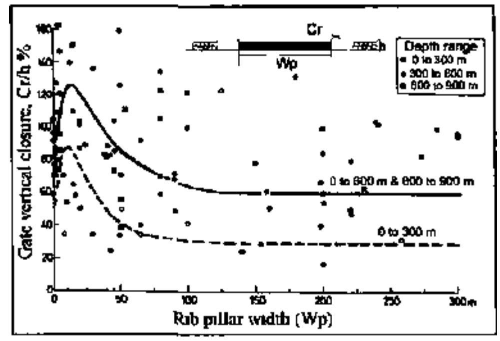

Figure 3 Statistical analysis of roadway closure for various pillar widths (Whittaker & Singh, 1979)

Holland (1973) studied the pressure arch concept for board and pillar mining and suggested that a pressure arch develops between the two barrier pillars in a panel when the pillars are designed to yield. İn this approach, the yield pillars are expected to redirect the overburden stresses to the solid abutments, thereby allowing greater extraction ratios within the panels. In the development stage of longwall mining, the pillar will be loaded by the tributary area theory. Where yield pillars are left under an intact pressure arch, the effective upper boundary of the tributary area is the bed separation limit. The pillar can be designed to yield either at the development stage or at the longwalhng stage. Figure 4a shows that the pillar is too wide to yield and yielding occurs at the ribs; the load carried by the yielded section is probably transferred to the inner pillar elastic core and to the abutment sides. In the second case (Figure 4b), the pillar is not proportioned so that it will retain sufficient flexibility and not pick up the full overburden load. The pillar load increases when die size of the entries is increased. However, tensile failure probability at the mid-span of the entry restricts the size of the entry. Failure of the pillar should occur İn a non-violent manner and it must maintain enough residual strength to support the weight of the rock within the pressure arch.

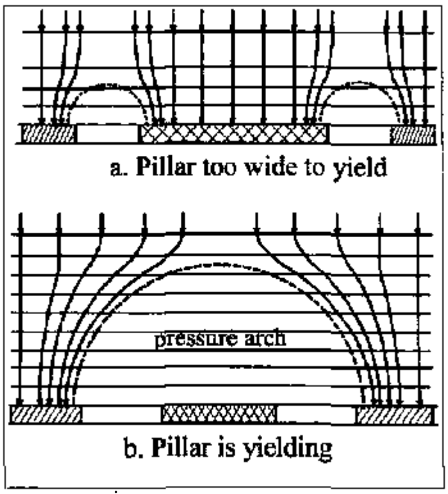

Figure 4 Redistribution of ground stresses for different pillar widths due to entry development

In the longwalling stage, cantilevered beds above the caved zone load the rib of the coal seam. These loads can cause excessive damage to the roadway depending on pillar behaviour. A theory was developed in the UK which states that a destressed zone develops inside an ellipsoid, while outside the ellipsoid the stress is high (Alder et al., 1951). The width of the ellipsoid 'b' shown in Figure 5 depends upon the cover depth. The vertical width of the ellipsoid is twice the horizontal width.

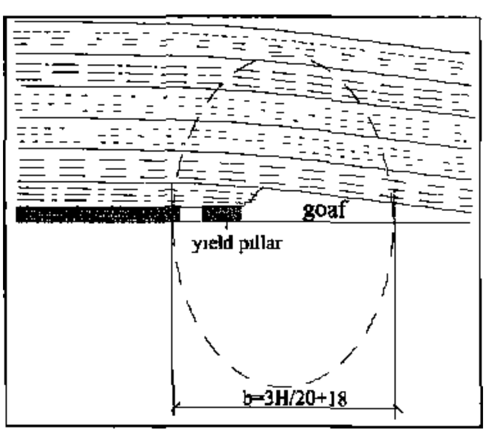

Figure 5 Pressure arch concept for longwalling stage of mining.

3 DESIGN OF A YIELDING PILLAR

A pillar design method should simply take into consideration that the width of a pillar should be adjusted so that yielding is ensured and that the width-height ratio should be adjusted so that it has high residual strength, maintaining the stability of the roadway. The failure and load-carrying capacity of a pillar depends on the intact and broken strength properties of the coal, the width-height ratio of the pillar, the rock properties and the properties of the rock-coal interfaces for a site-specific investigation. Current methods do not consider all of these factors. However, they can be applied to specific cases with the necessary experience in die field.

3.1 *Empirical methods*

In a two-entry system, the design of a yield pillar may be accomplished using empirical formulae and by determining a mine-specific safety factor through field experience. All empirically derived equations can be written in two types of expression.

$$S_p = S_i[\,A + B(w_p/h_p)\,] \tag{1}$$

$$S_p = S_i(w_p^a/h_p^b) \tag{2}$$

where S_p is die pillar strength in MPa, Si is the in-situ strength of coal in MPa, w_p is the width of the pillar in m, h_p is the pillar height m m and A, B, a and b are constants expressing the shape effect (Table 1).

Table I. Details of pillar strength equations.

Author	A	B	a	b
Obert& Duval! (1967)	0.778	0.222		
Holland (1973)	-	-	0.5	0 5
Bieniawski (1984)	0 64	0.36		
Salamon & Munro (1967)	-	-	0.46	0 66

The empirically derived equations given above can predict the overall strength of squat pillars. The strength of infinitely long and rectangular pillars can be significantly greater than that of square pillars due to the greater confinement generated within them. The strength of large rectangular specimens may be expected to be the same as that of large square specimens with a side length equal to the effective width of rectangular specimens. Wagner (1974) suggested the following equation for estimating the effective width of long pillars:

$$w_{eff} = \frac{4.A_{rp}}{U_{rp}} \tag{3}$$

where Arp is the area of the pillar and Upp is the pillar circumference. According to equation 3, the effective width of an infinitely long barrier pillar is twice the actual width.

The load coming over the pillar İs calculated using the tributary area method, which is commonly used in room and pillar mining. This load is called the development load and İs calculated as follows:

$$A_d = \gamma H\left(\frac{w_p + w_e}{w_p}\right) \tag{4}$$

where w_p is the pillar width (m), w_e is the entry width (m), H is the-depth (m) and y is the unit weight of the overburden (MN/m^3).

Empirical methods can be successful provided that correct selection of the safety factor, which is the ratio of pillar strength to stress imposed on the pillar, is made through field experience for each specific formula. A comparison was made for a depth of 600 m and a 2-m-thick seam. The strength equations give varying safety factors for the same pillar width as illustrated İn Figure 6. This difference İs more significant when the width of the pillar increases. Carr (1992) reported that safety factors in the range of between 0.52 and 0.73 were successful. The width of the pillar to yield ranges between 6 and 11.5 m. This range of widths İs in agreement with successful field applications, as shown below.

Although the empirical design of pillars gives an idea of the width-height ratio of the pillar where yielding is expected, it does not estimate the failure behaviour of a pillar that exhibits either gradual or sudden failure.

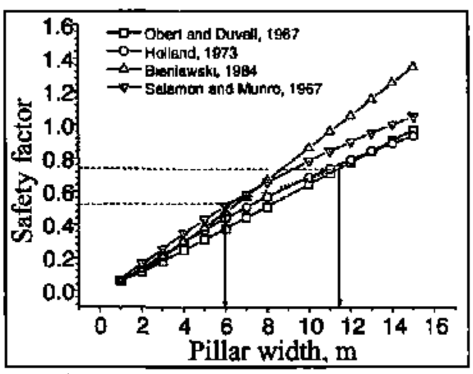

Figure 6. Yield pillar width ranges for 2-m pillar height.

3.2 *Mine stiffness concept*

In-situ tests performed by Wagner (1974) clearly showed that the failure of a pillar is a gradual process as shown in Figure 7. The distribution of the load across *the* cross-section of the pillar varies as the loading proceeds. Yield at the pillar edges and an increase in the load bome by the pillar core can be seen clearly. Wagner's tests proved conclusively that a pillar has a significant load-bearing capacity even when its maximum resistance, which is traditionally regarded as the strength of the pillar, has been overcome. This is the main area of interest in the design of yield pillars.

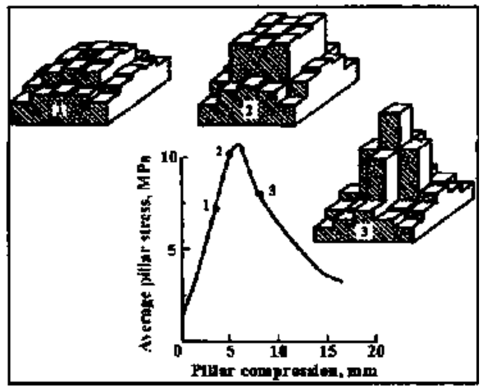

Figure 7. In-situ complete stress deformation curve and stress distribution of pillar with width/height ratio of I (Wagner, 1974).

The deformation characteristics of coal pillars were investigated by means of underground tests on large coal specimens which were carried out by Van Heerden (1975) in South African coal mines. The results of these tests indicated that the deformation characteristics, in particular the post-peak behaviour, of coal pillars are not only a function of die coal itself, but most importantly of the pillar geometry (Figure 8). These tests can be used for the estimation of stiffness estimation in pillars with width/height ratios of up to 3.5.

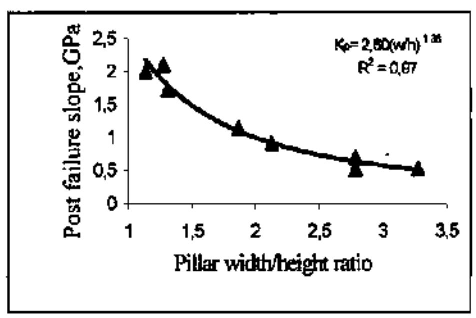

Figure 8. Effect of width/height ratio on the post-tailure slope of coal pillars (Van Heerden, 1975).

Bearing in mind the yielding mechanism of pillars, Salamon (1970) showed that equilibrium between the loading of a pillar and post peak pillar resistance is stable, regardless of the convergence experienced by the pillar if;

$$K_{LMS} - K_P < 0 \qquad (5)$$

where K^ is the stiffness of the loading strata and K_P is the minimum slope of the post-peak load-deformation relation for the pillar (both K^ and K_P have negative values). These cases are illustrated in Figure 9, showing that the relation between the stiffness of the strata and post-peak load deformation determines the stability condition of the pillar.

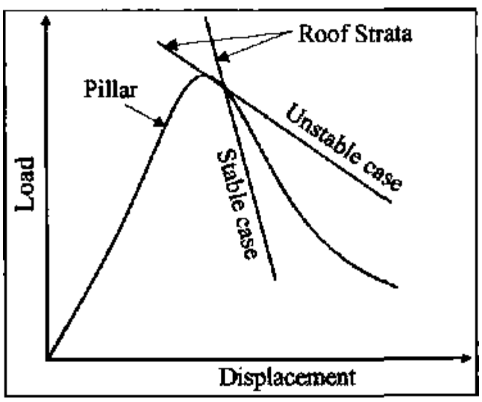

Figure 9 Stable and unstable cases for a pillar depending on stiffness of loading strata.

In assessing the stability of a mine structure, the required information consists of the post-peak stiffness of the pillar and die mine local stiffness at various pillar positions. The stiffness of the strata at an individual pillar location, local stiffness, is described as the load deformation between the hanging wall and footwall. The finite difference method was utilised to find the stiffness of strata at a pillar location at Bilsthorpe coal mine. The model conditions are illustrated in Figure 10. The average depth is 600 m. The deformation modulus of the rock mass is 14 GPa. The detailed input data for the Mohr-Coulomb plasticity model was described by Yavuz (1999).

The pillar was replaced by stresses for pillar widths of 5, 7.5 and 10 m as illustrated in Figure 10. The strains between the hanging wall and footwall at the pillar location are plotted against the stress applied on the roof (Figure 11). The post-failure slopes of the 5, 7.5 and 10-m pillars were found to be 0.73, 0.42 and 0.28 respectively from the equation given in Figure 8. If the slope values of the strata given in Figure 11 and the post-failure slope values are put into equation 5, an unstable condition will be expected for 5 and 7.5-m. pillar widths. This finding is questionable in terms of the predicted post-failure slopes for pillars with width/height ratios of 3.75 and 5 due to the unavailability of data.

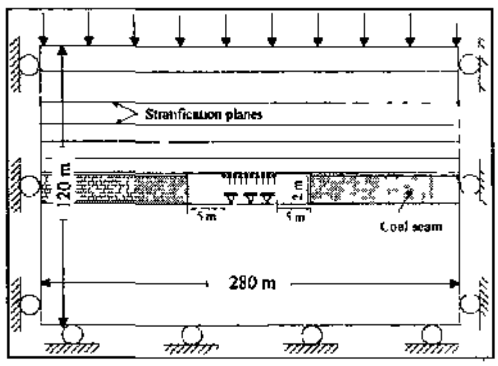

Figure 10. Modei conditions for mine stiffness estimation

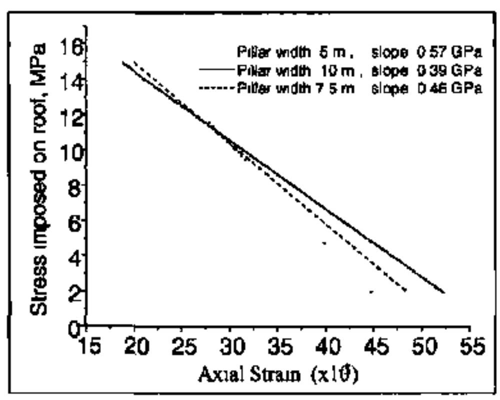

Figure 11 Stress & strain relation from roof to floor for 5, 7 5 and 10-m pillar widths

3.3 *Enhanced confined core method*

The friction between the loading platens and the specimen significantly affects the strength of the specimen under uniaxial loading conditions This will be a primary factor in the generation of confining pressure and increase in strength. The Wilson confined core concept considers confinement of the pillar; how ever, this method is based on an assumed horizont.il restraint and does not consider the level of confinement for varying properties of the coal-rock interfaces. It is a known fact that if the friction angle is reduced due to filling material, the strength of the coal is reduced.

Salamon (1992) developed an enhanced confined core concept which takes the coal-rock interface properties into account Failure at pillar edges depends upon the following criterion:

$$\sigma_{max} \geq kp + S_1 \tag{6}$$

where k= $(1+\sin\phi)/(1-\sin\phi)$, ϕ is the friction angle of the coal, p İs the pillar side restraint, S, is the in-situ compressive strength of the coal, and σ_{max} is the maximum stress at the edge of the fully elastic pillar, found by the following equation:

$$\sigma_{max} = q\left[1 + \frac{w_e'}{\theta h \tanh(w_p'/\theta h)}\right] \tag{7}$$

where q is the vertical virgin stress, $w_e = w_e / 2$, w_e is the entry width, $w_p = w_p/2$, w_p is the pillar width (Figure 12), h is the seam thickness, and the constant 6 İs given by the following relation:

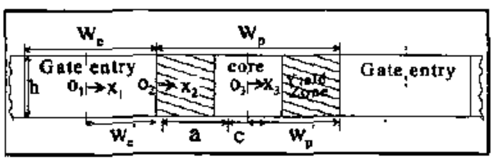

Figure 12 Geometry and notations for yield pillar design.

$$\theta = \left[\frac{(1-\upsilon_s - 2\upsilon_s^2)\lambda E}{2(1-\upsilon_s)hE_s}\right]^{1/2} \tag{8}$$

where E, andt), are the Young's modulus and Poisson's ratio of the seam. E is the Young's modulus of the rock. X is a constant related to the deformation of the surrounding strata found from $X= H / 2(\hat{u}^2$, (0 is a constant related to surface subsidence. Salamon suggested values for to of 5-7 for coal mining. If equation 6 satisfies the condition, the pillar edges will yield and the load-bearing capacity of the pillar is found as follows:

$$S_{max} = \frac{p}{\mu R_p}(e^{\eta R_p/2} - 1) \tag{9}$$

where R_p is the width/height ratio of the pillar, p. is the interface coefficient of friction(ti = tan(p), <p is the interface friction angle and

$$\eta = 2\tan\phi\left[\frac{\sin\varepsilon_m}{1-\sin\phi\cos\varepsilon_m} + 2\tan\phi\tan^{-1}\left(\frac{1+\sin\phi}{\cos\phi}\tan\frac{\varepsilon_m}{2}\right)\right] \tag{10}$$

$$\varepsilon_m = \sin^{-1}(\mu/\tan\phi) \tag{11}$$

The load on the pillar is estimated by using the tributary area method in the following way:

$$q_m = \frac{w_p + w_e}{w_p} q \tag{12}$$

If the load q_m exceeds the load-bearing capacity of die pillar S_{max}, then the pillar fails. If it does not exceed the load-bearing capacity, then the pillar edges will yield, but the pillars will be able to sustain the load.

The changes in the width of die pillar for failure were investigated for a depth of 600 ra, wheic S, is 5 MPa, γ is 0.025 MN/m^3, E is 10 GPa, E_s is 2.5 GPa, υ_s is 0.3, ω is 7, h is 2 m and w_e is 5 m. The width of the pillar was calculated. This width is given in Table 2 for different friction angles of coal and interface and for different side restraints.

Table 2. The width of the pillar for complete yield for various friction angles for (ϕ), bedding friction a n g (φ) n d edge restraints (p).

ϕ (deereet	φ (degree)	p(MPa)	Wp(m)
35	25	0.1	8.4
30	25	0.)	10.9
35	20	0.1	9.9
35	25	0.02	10.9

3.4 *Numerical method*

As mentioned above, a yielding pillar provides a destressed zone around the entries by transferring the stresses from over the pillar to over abutment sides. The numerical modelling technique, when compared to the other design methods, is a powerful method for demonstrating the yielding situation of a pillar and stress state over the working area provided that enough in-situ data are available to construct the models. Although most design methods ignore the stress distribution within the pillar and interaction between the roof, pillar and floor, these data can be taken into consideration in numerical models. Parametric studies and field applications showed the importance of these factors in the design of yielding pillars. Numerical models using a two-dimensional finite difference code, FLAC, were arranged for investigation of the stress magnitudes over the pillar under the influence of depth stress and front abutment stresses. The boundary conditions of the models are the same as those given in Figure 10. A strain-softening model constituted the post-failure behaviour with widths of 6, 7.5 and 10 m for the model pillars. By applying back analysis to the data available from the large-scale tests of Van Heerden (1975), the friction angle and cohesion of the yielded coal were found to be 23° and 0.35 MPa. The in-situ compressive strength and deformation modulus of the coal mass are 4.78 MPa and 2.5 GPa. The rock mass surrounding the working area is quite strong. Bedding planes, illustrated in Figure 10, were also modelled and the friction angle between the coal-rock interface is 27 degrees.

The distribution of vertical stresses immediately above the pillar, gate entry and abutment are illustrated in Figure 13. In this model, it is assumed that the Iongwall face is far enough for building up front abutment stress over the modelled section. The stresses are illustrated for just a half side of the pillar due to its symmetry. The findings from these models are that a 6-m pillar yields and retains a small amount of stress, about 10 MPa, at the centre of the pillar, while 7.5 and 10-m pillar rib sides yield. However, the pillar core retains a significant quantity of stresses.

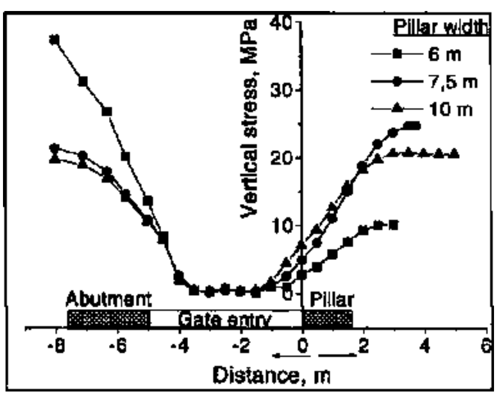

Figure 13. The stress state over the wotting region and pillar at a depth of 600 m with no front abutment stress.

The vertical stress was gradually increased over the vertical upper boundary of the models in order to represent the front abutment stresses over the modelled section. The stress distribution over the modelled section illustrated in Figure 14 is just for a 25-MPa boundary stress condition. In this case, no increment occurred over the 6-m pillar; excess stresses were transferred to the abutment side. It is interesting to note that the 7.5-m pillar failed gradually with the increment of front abutment stresses and the stress at the centre of the pillar decreased from 25 MPa to 19 MPa. The 10-m pillar width can be regarded a critical width for the properties assigned to the model.

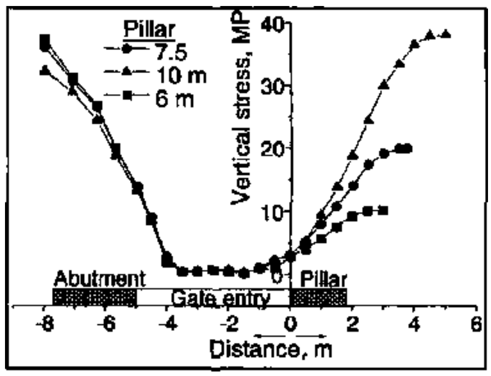

Figure 14. The stress state over the working region and pillar at 600 m depth with front abutment stress

Parametric studies showed that the properties of the pillar-rock interface, as well as the softening properties of die coal material, significantly influence the yielding state of a pillar and stresses over the pillar and working area.

3.5 *Field experience and stress measurements*

The difficulty in stress measurement is the reliability of the obtained values. However, the evaluation of stress measurements and observation of roadway stability is a practical method of site-specific design, especially for yield pillars. The width of the pillar to yield during the development loading or the side abutment loading stage ensures improved ground conditions for pillars between 6 and 10 m wide, depending on the coal mass strength, seam height, roof and floor constraint and mining depth. The application of these sizes is generally successful in deep mines (Table 3). However, a 9-m-wide pillar failed to yield properly under 365 m of cover load, and at some mines, the changing of a proven successful design to slightly larger 12-15-m yield pillars under deeper cover seems to have resulted in renewed ground control problems, both in the roof and floor (Demarco et al., 1988).

4 CONCLUSIONS

For a pillar to be called a yielding pillar, it must display gradual yielding, not sudden failure, and maintain enough residual strength to support the weight of the rock within the pressure arch during

Table 3. Yield and critically sized pillar widths at various depths in the field.

Pillar width (m)	Depth (m)	Yield condition	Pillar core stress (MPa)	Face position (m)	Reference
16.8	795	critical	65	0	Ko*leraaL1996
12.2	580	yield	20	-24	KœhkyetaU996
10.6	855	yield	-	-	KœhfcxetaU996
9	460	yield	12	0	Demacoaall988
9	460	yield	14	0	Danacoetall988
6.1	610	yield	7	76	Newiml989
10	840	yield	10.5	25	HadnetaL1997

*(0 longwall race is approaching measurement point

the development and side abutment loading stages of mining. The current design methods introduced in this study are not unique themselves to explain the yielding mechanism of these pillars. All the methods used for the preliminary estimation of long yielding pillar width ranges for Bilsthorpe Colliery estimated a reasonable width range when compared to the previous applications. The main conclusions of this study can be summarised as follows:

1. Empirical equations with regard to the tributary area method predicted a width range for a yielding pillar of between 6 and 11.5 metres for Bilsthorpe colliery by considering safety factors between 0.5 and 0.73. The ultimate strength concept, of course, does not consider the residual strength of the pillar, interaction between die roof, pillar and floor, and the stress distribution within the pillar. However, diese factors can be taken into account by determining a site-specific safety factor. This means that generally the predicted width ranges of the empirical equations are acceptable based on field experience.

2. The mine stiffness concept has two weaknesses in predicting the stability of a yielding pillar. Firstly, there are not enough data available for the post-failure slopes of large-scale pillars. Secondly, it does not take the pillar-roof interface properties into consideration. The predicted post-failure slopes and calculated stiffness values from the numerical models suggested that a 10-m pillar could be regarded as a yielding pillar, while the 5 and 7.5-m pillar widths were found to be unstable in a two-entry system. This finding is true for the 10-m pillar; however, the 7.5-m pillar width in the two-entry system is a yielding pillar in some mines.

3. The confined core concept is a valuable analytical method in comparison to the other analytical methods in which no account is paid to the properties of coal-rock interfaces. However, pillar side restraint is based on an assumed value which strongly affects prediction of the yielding width of a pillar. There İs no attention paid İn this method to the post-failure properties of the pillar.

4. Numerical modelling has certain advantages when compared to the other methods since it takes the main factors into account in the yielding process of the pillar. However, the prediction and reliability of results in mis method depend on the availability of in-situ data. For a pillar height of 2 m, the predicted width of the yielding pillar for the properties assigned to the models is 6 m, without considering the front abutment stress. The 7.5-m pillar can also be regarded as a yield pillar. The 10-m pillar, which was determined for this example mine as a critical width, could be successfully applied in other mines, depending on the shear strength properties of rock-coal interfaces as well as the softening properties of the coal material.

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