

Aspects Regarding the Mathematical Modeling of Stress- strain for Mining Works

V. Arad & S. Arad
Petrosam University

NIanc
INSEMEX Petroiam, Romania

ABSTRACT: The paper presents the stress and strain state around a tunnel lining applying the confining and convergence method, used some time to dimension tunnel linings

This method can be developed in an analytical form only on restrictive hypotheses for the tunnel geometry and the initial stress of the rock. A fundamental aspect is applying the convergence - confinement method is represented by the assessment of massif convergence when the lining is activated and which depends by the completing of the lining performance when it starts to introduce a pressure to oppose to the massif convergence

1 INTRODUCTION

The mining activity performed in Romania until the year 1990 was focused mainly on reaching high production levels, fact which had led to neglect ecological issue and that resulted in time, very serious damages for the environment. The technical depletion and the decrease of governmental subventions determined down of working productivity.

Simple observations indicated that the horizontal stress regime at the mine was relatively benign.

These indications included the lack of any stress indications at the head end, little or no floor lift and the stability of roadways, both from the point of view of roof and floor closure. Negative aspects included consistent over height with material dropping out and the apparent rapid weathering of the sides.

2 THEORETICAL CONSIDERATION

The design of the mining works linings as permanent retaining structures, taking into account the lining-ground interaction is influenced by the geological and physical - mechanical characteristics of the rock massif, the physical - mechanical characteristics of the lining and their time dependency (Arad, 1995).

The final equilibrium of the lining is given by the intersection of the rock confining curve in the excavation walls and the lining convergence curve and it can be reached differently depending on the time of lining performance. The behaviour domain of the massive rock where the tunnel of circular cross - section is performed is elastic and then elastic - plastic with the same restriction concerning the initial stress state.

The convergence - confinement method represents an analytical method that gives the possibility to take into account different factors which influence the final equilibrium of the rock massif - tunnel lining system like in papers (Panet, 1995, Bud, 2001)

- The tunnel deformability - (time dependency behaviour before yielding),
- The lining stiffness,
- The massif deformability before performing the lining

Four specific domains (figure 1) define the elasto - plastic behaviour of soils and hard rock

- 1 An elastic behaviour domain, until the elasticity limit (I),
- 2 A plastic strain domain limited by the maximum strength (II),
- 3 A plastic strain domain with consolidation limited by the residual strength (III),
- 4 A domain of perfect strains in the conditions of residual strength (IV)

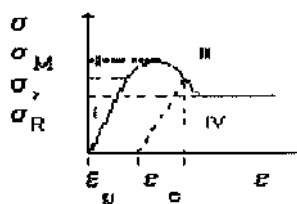


Figure 1 The domains of elasto plastic behaviour of soils and hard rocks

3 THE CONVERGENCE LAW FOR THE CASE OF AN ELASTO-PLASTIC BEHAVIOUR OF THE MASSIF (PANET, 1995)

The ti' A\A excavation induces stress redistribution within the massif (Arad, S; Arad, D; 2000). If it is considered the rock massif with perfect elasto-plastic behaviour, subjected to the Mohr - Coulomb failure criterion, one may write the following relationship:

$$f(\sigma_1, \sigma_3) = \frac{\sigma_1 - \sigma_3}{2} - \frac{\sigma_1 + \sigma_3}{2} \cdot \sin \varphi - 2c \cdot \cos \varphi = 0 \quad (1)$$

or:

$$f(\sigma_1, \sigma_3) = \sigma_1 - j \cdot \sigma_3 - Q = 0 \quad (2)$$

where:

$$j = \frac{1 + \sin \varphi}{1 - \sin \varphi}; \quad Q = \frac{2C \cos \varphi}{1 - \sin \varphi},$$

one dimensional compression strength.

Considering the axis-symmetrical case, the stress redistribution in the plastic zone is obtained by integration of the equilibrium equation:

$$\frac{d\sigma_r}{dr} = \frac{\sigma_r - \sigma_\theta}{r} \quad (3)$$

If the rock massif begins to work in the domain of linear elastic behaviour, the radial stress σ_r , and the orthoradial stress, σ_θ , on the tunnel wall of circular cross - section are given by relationships:

$$\sigma_r = (1 - \lambda) \cdot \gamma_a H \quad \text{and} \quad \sigma_\theta = (1 + \lambda) \cdot \gamma_a H \quad (4)$$

where: λ represents the confinement degree the failure conditions are reached at the wall border for a value λ_c of the confinement degree, in which case one may write:

$$f[(1 + \lambda_c) \cdot \gamma_a H, (1 - \lambda_c) \cdot \gamma_a H] = 0 \quad (5)$$

where:

$$\lambda_c = \frac{1}{1 + j} \left[j - 1 + \frac{\sigma_c}{\gamma_a H} \right] \quad (6)$$

For $\lambda < \lambda_c$ the plastic zone does not appear and for $\lambda > \lambda_c$ a plastic zone is developed around the tunnel with a radius that increases with the confinement degree.

4 THE STRESS AND STRAIN STATES FOR THE CASE OF AN ELASTO -PLASTIC BEHAVIOUR OF SOIL

The strain increase in the plastic zone equals the sum of the elastic and plastic strain increases:

$$\varepsilon = \varepsilon_e + \varepsilon_p \quad (7)$$

If it is considered that the elastic strain is negligible in comparison with the plastic strain, the main strains verify the relationship:

$$K \cdot \varepsilon_1 + \varepsilon_2 = 0 \quad (8)$$

Where:

$K = 1$, when the strain is produced at constant volume;

$K > 1$, when the strain is produced at the volume an increase.

The radial displacement in the plastic zone is given by the integration of the differential equation (Boti, N; Stanciu, A; Droniuc, N; Lungu, I, 2000):

$$\frac{du}{dr} + K \cdot \frac{u}{r} = 0 \quad (9)$$

In the conditions of isotropic initial stress ($K_0 = 1$) solving the equation goes to the following:

$$\sigma_1 = \sigma_\theta; \quad \sigma_3 = \sigma_r \quad (10)$$

In the limit conditions:

$$r = a; \quad \sigma_r = 0; \quad r = \infty; \quad \sigma_r = \sigma_\theta = \gamma_a H \quad (11)$$

The differential equation for the axis-symmetric case becomes:

$$\frac{du}{dr} + K \cdot \frac{u}{r} = \frac{1 - \mu^2}{E} \left[\left(K - \frac{\mu}{1 - \mu} \right) \Delta \sigma_\theta + \left(1 - \frac{\mu}{1 - \mu} \cdot K \right) \Delta \sigma_r \right] \quad (12)$$

where: $\Delta \sigma_r = \sigma_r - \gamma_a H$, and $\Delta \sigma_\theta = \sigma_\theta - \gamma_a H$.

The obtained solution by the integration of this equation is the following:

$$u = \lambda_e \cdot \left[F_1 + F_2 \left(\frac{r}{b \cdot p} \right)^{j-1} + F_3 \left(\frac{b \cdot p}{r} \right)^{j+1} \right] \frac{r \cdot \gamma_a \cdot H}{2G};$$

where

$$F_1 = -(1 - 2\mu) \cdot \frac{j+1}{j-1};$$

$$F_2 = 2 \frac{1 + K \cdot j - \mu \cdot (j+1) \cdot (j-1)}{(j-1) \cdot (K+j)};$$

$$F_3 = 2 \cdot (1 - \mu) \cdot \frac{j-1}{K+j};$$

(13)

The radius of the plastic domain results by the relationship:

$$b_p = a \left[\frac{2\lambda_r}{(j+1)\lambda_r - (j-1)\lambda} \right]^{\frac{1}{j-1}} \quad (14)$$

where: a - radius of the tunnel.

5 THE USE OF THE NUMERICAL MODELS TO APPLY THE CONVERGENCE - CONFINEMENT METHOD

In the convergence - confinement method the plane strain problem is the substitute of the three - dimensional problem, where the following stress is applied at the outer lining.

$$\sigma_r = (1-X)Y_a H \quad (15)$$

corresponding to the appropriate confinement for the considered construction stage.

The study of the massif - tunnel lining interaction that develops during the tunnel excavation, applying the convergence - confinement method includes generally the following stage:

Stage 1 - the construction at the finite element network of the model.

Stage 2 - the introduction of the initial equilibrium conditions, obtained the model forces F_{\dots} , on the inner tunnel lining.

Stage 3 - the simulation of the tunnel excavation and lining performance; the nodal forces that act at the inner lining are decreased at the value $X V_{gr}$, where X is the confinement degree which corresponds to the excavation progress before the performance of the lining.

Stage 4 - the simulation of the lining performance by activating the appropriate finite elements.

A fundamental aspect is applying the convergence - confinement method is represented by the assessment of massif convergence when the lining is activated and which depends by the completing of the

lining performance when it starts to introduce a pressure to oppose to the massif convergence.

To determine the confinement degree, X_c the method based on the convergence curve of the unsupported tunnel and implicit methods can be used. Based on previous studies, it was noticed that if the confinement degree is almost the same, 0,03, for an elastic behaviour of the rock massif, for elastic - plastic behaviour there is no explicit solution and it is recommended to use the successive approximation method.

6 RESULTS AND EXPERIMENTS

Based on the previous theory and taking into account that deformation is influenced by a large number of factors, we analysed the stress- strain state of a horizontal mine workings

The geostatic forces increase with the depth and play a significant role in the analysis of the rock mass deformation. These forces together with the tectonic ones cause remaining stresses which cause the modification of geomechanical properties.

From the stress state analysis of the rock mass, present the maximum stress variation on a mine working contour line span $2a = 4.5$ m and $h = 3.7$ m, located at various depths for some rock types , prevailing in the Jiu Valley Coal Basin, table no. 1.

The stresses acting around an underground working can be determined analytically or experimentally by measurements, Table no.2.

For the above mentioned mine working located in argillaceous rock having the following characteristic: $E = 1,5 \cdot 10^3$ [MPa], $\nu = 2.57 \cdot 10^3$ [N/m²] and $\mu = 0.29$, by applying the finite element method, the results have been obtained that are in agreement with those determined experimentally and they are rendered in Table 2.

Table 1 The maximum stress variation on diverse depths for some rock types from Jiu Valley Coal Basin

Rock type	Maximum stress value for the depth variations							
	100 (ml)	200 [m]	300 (ml)	400 (ml)	500 (ml)	600 (ml)	700 (ml)	800
Chocolate clay	5.77	121	18.4	24.7	31.2	35.8	42.1	48.6
Siliceous sandstone	7.33	146	22.1	29.8	36.9	44.2	52.2	60.7
Marl	6.25	13	196	26.1	33.1	40	45.7	52.8
Doad	641	128	191	26.3	32.9	39.2	45.8	53.1
Arguaceous sandstone	7	14.1	21.1	28	35.1	42.1	48.8	55.9

Table 2. The values of maximum stress determined

Maximum Stress σ_0 [MPa]	Depth H [ml]							
	100	200	300	400	500	600	700	800
Analytically	5.7	12.1	18.4	24.7	31.2	35.8	42.1	48.6
Numerically	5.9	13.1	19.5	25.8	32.1	36.4	43.0	49.8

7 CONCLUSIONS

An important aspect in the construction of mining works is ensuring that they function properly. The

V Arad S Arad & N lane

underground constructions in general unfortunately can be a negative effect on surface or have been in future disturbed by natural factors of the environment

In the paper a numerical model of the rock-support interaction for tunnels by the underground construction is determined

This study has been performed for the two-dimensional stress and strain state

The use of the convergence - confinement method in the rock massif - lining tunnel interaction in a circular tunnel may be also realised for an elasto - plastic behaviour of the rock

The stress state is obtained for both rock massif and tunnel lining and based on these results the behaviour in service is predicted for the tunnel that is to be performed

REFERENCES

- Arad, D , 1995 Mining Geotechnics Technical Publishing House Bucureşti
- Arad, S, Arad, D , Chindns, Gh, 2000 Environmental Geotechnic mediului, Polodava Publishing House Deva)
- Boti, N, Stanciu, A, Dronmc, N, Lungu, I, 2000 Numerical Modeling of the rock support interaction for circular tunnels, in *Proceedings of the third National Conference of Undei ground Constructions 2000*, Petrosani, pp 32-38
- Bud, I, 1999 Studiul regimului de presiune în lucrările miniere de pregătire penini exploatarea zăcămintelor filoniene în scopul perfecționăm sustineni la minem dm bazinul Baia Mare PhD thesis (unpublished), Petrosani University, Petrosani
- Garbea, D 1990, *Analim eu elemente finite* , Bucureşti,
- Panet, M ,1995 Calcul des tunneles par la méthode convergence - confinement (Presses de l'Ecole Nationale des Ponts et Chaussées Paris)
- Stematiu, D, 1988, *Calculul structurilor hidrolehnice prin metoda elementelorfinite* Bucureşti
- Zenkievici, O , Gang, I, 1974, *Metod konecimh elementov v teorju soorujenu i v mehamka sploymh sred*, Moscova Nedra