Inelastic and Damage Modeling of Tunnel Face Surrounded by Discrete Elements

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ABSTRACT: The discrete elements become more and more popular in describing damage and inelastic behavior of plenty of materials. Free hexagons are used as special discrete element method to describe nucleation of cracks in soil, or rock. The problem is aimed to the stability of tunnel face. The method proposes a continuation and development of Desai's DSC (distinct state concept), which has formerly been used for saturated soils. Together with Transformation field analysis they create a powerful tool for assessment of tunnels.

1 INTRODUCTION

The neighborhood of the tunnel face is discretized into discrete elements, which are connected by fictitious springs simulating clay holding stick or causing disconnections between the elements (Prochazka 2004). The discrete elements are considered as free hexagons, their origin is in finite element method (Onck & van der Giessen 1999). Dynamical version of discrete elements can be found in (Cundall 1971), who starts with dynamical equilibrium of balls. This concept seems to be very appropriate for earthquake problems, but for almost statical problems is less advantageous. The stresses cannot be expressed, for example.

Models are considered for application to plasticity, viscoelasticity and damage in soil/rock material. Model by (Kachanov 1992) is mostly used. The method proposes a continuation and development of Desai's DSC (distinct state concept) which has formerly been used for saturated soils (Desai 1994) and extended to solid materials (Desai 1994, Moreau 1994). The extension by Desai consisted in inclusion of skeleton into the consideration and solution of such problems, which were coupled (mutual interaction of water and skeleton was studied). Prochazka and Trckova (Prochazka & Trckova 2000) introduced previously piecewise uniform eigenstrains in each material phase and precised the properties of the phases. Standard applications of the method to a two-phases rock material (stone, clay) are considered in this study, it means only one sub-volume per phase is considered. Discontinuous model is used by the discrete elements with softening respecting exclusion of tensile stress overstepping the tensile strength in springs connecting the elastic elements (Prochazka 2004). In the same time the shear cracking occurs in the tangential direction of the possible crack, which are considered in the shear springs and they make disconnections in displacements. The main disconnection is due to tensile stress in normal direction (in the springs being oriented in normal direction). In this case, the discontinuous models can be used with more promising future. Because those discrete models can describe the situation more realistic, they are worked out in more details in (Prochazka & Trckova 2001). A typical application of coupled modeling (experimental and numerical) to special case of patterns was published in (Trckova & Prochazka 2001). We hold the discontinuous model and substitute the slip caused by overstepping the damage law by introducing generalized Mohr-Coulomb law on the interfacial boundary. The different aspects of the proposed methods are systematically checked by comparing with finite element unit cell analyses, made through periodic homogenization assumptions, for three-directional lay-ups.

In order to complete the accuracy of a design of tunnel stability, Transformation field analysis (TFA) is used (Dvorak & Prochazka 1996). This is a generalization of original work by (Dvorak 1992). Another application of TFA is published in (Dvorak et al. 1999), where optimal design of bearing capacity of submersibles was solved. All the above-mentioned works on TFA are concerned with uniformly distributed eigenparameters. (Michel & Suquet 2003) extended the assumption of uniformly distributed eigenparameters to nonuniform transformation field analysis.

Starting with certain mechanical model (Desai's model in our case) the TFA brings about more accurate description of the physical conditions.

2 GENERAL CONSIDERATIONS

There are main different methods and tools that can be used to deliver the macroscopic constitutive response of heterogeneous materials from a local description of the microstructure behavior. Here we are concerned with non-linear behavior by the inelasticity of constituents or with the initiation and growth of damage. In the development of the homogenization procedures for non-linear materials we have to define both the *homogenization step* itself (from local variables to overall ones) and the often more complicated *localization step* from overall controlled quantities to the corresponding local ones.

The nonlinear problems of localization and homogenization are of a great importance today. Not only classical composites suffer from deterioration of the material due to hereditary problems (aging, viscoelasticity). On the other hand, composite materials prepared in a special way can improve properties of other material and the resulting effect can be much better than before. In this case, nonlinear and time dependent behavior has to be taken into account. From the wide scale of papers name here

The scope of the present paper is the development of constitutive equations for inelasticity and damage of heterogeneous materials that benefit from some specificities of a special boundary element method. On one hand, we need to obtain better approximations of the local stress and strain fields than in the Suquet based approaches, especially when considering damage and failure conditions. We want to simplify sufficiently the numerical techniques of overall homogenization in order to obtain a treatable system of equations that could recover the status of a constitutive equation.

The computation was run on Pentium IV PC, 2.6 GHz in FORTRAN. The program for generations of hexagonal of hexagon meshes of internal cells as well as the boundary nodes had been prepared using our own software. According to wish of the user, the meshing can be improved. The consumption of time for computation of even large system of equations, which can be stored into memory without use of hard disc or extended/expanded memory, was negligible in each step. Our illustration does not reach such dimensions of computation. It is also not necessary for such problems to increase the precision of the meshing, it losses the efficiency. The iteration at each step of loading was also very fast. It is worth noting that similar computations was carried out by the FEM, but finer meshing had to be imposed to get the comparable result with the BEM in the procedure presented. The comparison has been tested in such a way that the sum of the concentration factors should be the unit tensor.

In this contribution we are going to present the fundamental ideas of a numerical procedure leading to overall viscoelastic and damage behavior of rock matrix in a rock - tunnel lining aggregate. Based on the numerical models and mechanical laws it is possible to obtain the strain and stress in opposite to classical PFC, where no stresses are reasonably be reachable from the model (dynamical equilibrium is the starting point of balls, describing the original continuum).

Very important property of the above procedure is the non-linearity of the problem, which, when using some smart algorithm, can be solved by very powerful iterative process.

3 FREE HEXAGON ELEMENT METHOD

The free hexagon element method may be considered the discrete element method (DEM). The great disadvantage of some classical DEM, however, is the difficulty to feed them with material properties provided from laboratory tests (this is also the case of the particle flow code (PFC) (Cundall 1971), as the balls in (Cundall 1971) are connected by springs, while laboratories provide completely different material parameters being valid for continuum). This is here overcome by considering the material characteristics, which are similar to the continuum. The principal idea of classical DEM is adopted, and the domain defining the structure continuum is, in our case, covered by the hexagonal elements, and other then elastic material properties can be introduced, such as elastic-plastic, visco-elastic-plastic, etc. This step avoids the necessity to estimate the material properties of springs, which are essential, e.g. for PFC. The free hexagon element method fulfills a natural requirement due to the fact that the elastic properties are assigned to the particles, and other material parameters (angle of internal friction, shear strength, or cohesion) to the contacts of the elements. Since most particles are of the same shape it is possible to apply very powerful iteration procedures, because the stiffness matrix can be stored in the internal memory of a computer.

The computational model is described in this paragraph, where the relations needed for numerical computation are also introduced. The interface conditions are formulated in the next paragraph, where the Lagrangian principle is based on the penalty method. The penalty parameters are represented by spring stiffnesses; the springs connect the adjacent elements. The material characteristics of springs can possess a large value to ensure the contact constraints. On the other hand, if, say, the tensile strength condition is violated, the spring parameters tend to zero, and in this case naturally no energy contribution in the normal direction to the element boundary appears in the energy functional. This process excludes the possibility of a multivalued solution, and uniqueness of the solution of the trial problem is ensured. If we cut out the springs when a certain interfacial condition is violated, the problem turns to singular and has not unique solution. Then the way on how the particles move in some later stages of destruction of the trial structure cannot be described.

The hexagonal particles are studied under various contact (interfacial) conditions of the grain particles (elements). In this paper two contact conditions are considered:

- the generalized Mohr-Coulomb hypothesis, with exclusion of non-admissible tensile stresses along the contact (a rock mass, for example),
- limit state of shear stresses and exclusion of tensile tractions along the contact (a brittle coal seam, for example).

The first case is generally connected with applications in matrices of composite materials, of shotcrete, and overburden of tunnels, etc., and the second case is more appropriate for applications in underground seams. A two-dimensional formulation and its solution have been prepared and is studied in this paper.

The problem formulated in terms of hexagonal elements (which are not necessarily mutually connected during the loading process of the body, because of nonlinearities arising due to the interfacial conditions) enables us to simulate nucleation of cracks and their propagation. The cracking of the medium can be described in such a way that the local damage may be derived. Local deterioration of the material is also shown in the pictures drawn for particular examples. Such a movement of elements and change of stresses probably cannot be obtained from continuous numerical methods.

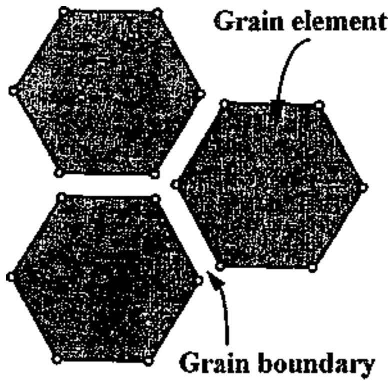

Figure 1. Geometry of adjacent hexagonal elements

3.1 Computational model

Let us now consider a single hexagonal element (described by domain Q with its boundary r). Its connection with the adjacent elements is shown in Figure 1. In each hexagonal element, the pseudo-elastic material properties are taken into consideration, i.e., in every iteration steps the element behaves linearly, but the material properties can change during the process of loading and unloading. This makes it possible to introduce only an elastic material stiffness matrix, which is homogeneous and isotropic, and we get well-known integral equations that are valid along the boundary abscissas of the hexagons, (Bittnar&Sejnoha 1996):

$$\sum_{l=1}^{2} c_{kl}(\xi) u_l(\xi) = \sum_{s=1}^{6}\Big(\sum_{i=1}^{2}\Big(\int_{\Gamma_s} p_i(x) u_{ik}^*(x,\xi)\,dx - \int_{\Gamma_s} u_i(x) p_{ik}^*(x,\xi)\,dx\Big) + \sum_{i=1}^{2}\int_{\Omega} b_i(x) u_{ik}^*(x,\xi)\,dx\Big), k = 1, 2, \quad (1)$$

where b_i are components of the volume weight vector, r, are edges (abscissas) of the boundary elements, <f is the point of observation, x is the integration point, «,. are components of the vector of displacements (defined not exclusively on the boundary, but also in the domain of the hexagonal element), p_t are components of the tractions, c_u is the matrix, the values of which depend on position of

the point of observation. The quantities with an asterisk are the given kernels. The kernels can be expressed as (see (Bittnar & Sejnoha 1996), for example):

$$u_{ik}^{*} = A(M\delta_{ik} (\log r - \frac{\bar{x}_i \bar{x}_k}{r^2}),$$

$$p_{ik}^{*} = -2A\frac{\mu}{r^2}(k(n_k \bar{x}_i - n_i \bar{x}_k) - (k\delta_{ik} + \frac{2\bar{x}_i \bar{x}_k}{r^2})\bar{x}_j n_j),$$

where

$$A = -(\lambda + \mu) / 4\pi\mu(\lambda + 2\mu),$$

$$M = (\lambda + 3\mu) / (\lambda + \mu),$$

$$k = \mu / (\lambda + \mu),$$

$$\bar{x}_i = x_i - \xi_i,$$

$$r^2 = \bar{x}_1^2 + \bar{x}_2^2,$$

and λ and μ are Lame's material constants.

Assuming uniform distribution of the boundary quantities (displacements $u_i(x)$ and tractions $p_i(x), i = 1,2,$ and volume weight forces b_l to be uniform in the domain Ω, and positioning the points of observation ξ successively at the points x_s, which are the centers of the boundary abscissas of the hexagonal elements, a simplified version of (1) is written as:

$$\frac{1}{2}u_k^s = \sum_{s=1}^{6}(\sum_{i=1}^{2}(p_i^s \int_{\Gamma_s} u_{ik}^{*}(x,\xi_s)dx - u_i^s \int_{\Gamma_s} p_{ik}^{*}(x,\xi)dx) + \sum_{i=1}^{2} b_i \int_{\Omega} u_{ik}^{*}(x,\xi)dx), k = 1, 2, \quad \textbf{(2)}$$

where u^* and p' are the values of the relevant quantities positioned at the ξ_s, s = 1,...,6, i.e., $u_i^s = u_i(\xi_s)$ and $p_i^s = p_i(\xi_s)$. Moreover, the vector of influences of the volume weight forces on the boundary abscissas is $b_s = (y_1, y_2), s$ = 1....6, and

$$\gamma_k^s = \sum_{i=1}^{2} b_i \int_{\Omega} u_{ik}^{*}(x,\xi_s)d\Gamma(x), \qquad £ = 1,2.$$

For better and more convenient computation, the most important integrals can be calculated in advance. In this way, the integrals in (2) may be calculated directly, without numerical integration.

Let us introduce vectors α_s, β_s, s = 1,...,6, and also u and p as:

$$\alpha_s = \begin{pmatrix} u_1^s \\ u_2^s \end{pmatrix}, \beta_s = \begin{pmatrix} p_1^s \\ p_2^s \end{pmatrix}, \mathrm{u} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{pmatrix}, \mathrm{p} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix}, \mathrm{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{pmatrix}$$

$$\alpha_k^s = \sum_{i=1}^{2}(-u_i^s \int_{\Gamma_s} p_{ik}^{*}(x,\xi_s)dx),$$

$$\beta_k^s = \sum_{i=1}^{2}(p_i^s \int_{\Gamma_s} u_{ik}^{*}(x,\xi_s)dx).$$

Using this notation, the relations on the elements (2) can be recorded as:

A u = B p + b **(3)**

where A and B are (12 * 12) matrices, and their components are singular integrals over the boundary abscissas. Matrix A is generally singular, while matrix B is regular. This fact enables us to rearrange equations (3) into the form:

Ku = p + V, K = B-'A, V = B-'b **(4)**

where the stiffness matrix K is different from that arising in applications of finite elements (here it is prevailingly non-symmetric), V is the vector of volume weight forces concentrated on the boundary abscissas (more precisely at the point ξ. In this way, the discretized problem becomes a problem similar to the FEM.

Along the adjacent boundary abscissas it should hold (p_i^μ are Eshelbys' forces):

$$p_i^+ + p_i^- = (p_i^\mu)^+ + (p_i^\mu)^-, \quad (5)$$

where superscript plus means from the right and minus from the left (at most two particles can be in contact).

Now using the relations (4) and (5), we get twice as many unknowns as equations, because no connec-

tion between the elements has yet been introduced. Equations (5) have to be accomplished by a constraint of the type

$$k_t(u_t^- - u_t^+) = p_t . \qquad (6)$$

The latter conditions are penalty-like conditions, since if k_t is great enough, the distribution of displacements is continuous, and the displacement from the right is equal to the displacement from the left. These conditions can locally be violated, because of the contact conditions, which are discussed later in this text. Introducing boundary conditions and assuming that k_t remains great enough leads us to a stable system of equations delivering a unique solution. Even in the case when local disturbances occur, the solution can be stable. It can happen that there are too many disturbances, e.g., dense occurrence of crack, and localized damage along a path (earth slope stability violation). Then the solution is unstable, and there is a failure of the structure. This is also, for example, the case of a behavior of composites.

Discretization in the previous sense leads to a nonlinear system of algebraic equations, which are solved by an over-relaxation iterative procedure. This method is sufficient for study purposes. For a larger range of equations the conjugate gradient method has been prepared.

For displacements in the element domain £2 it holds:

$$u_k(\xi) = \sum_{s=1}^{6}\Big(\sum_{i=1}^{2}\big(p_i^s \int_{\Gamma_s} u_{ik}^*(x,\xi)dx -$$

$$- u_i^s \int_{\Gamma_s} p_{ik}^*(x,\xi)dx\big) + \sum_{i=1}^{2} b_i^s \int_{\Omega} u_{ik}^*(x,\xi)dx\Big), k = 1, 2, \quad (7)$$

where the element boundary displacements and tractions are known from the previous computation, providing the solution is stable. Using kinematical equations and Hooke's law, the internal stresses can be calculated from (7). There is no danger of singularities, as the points x and ξ never meet (point ξ lies inside the domain Ω and x on boundary Γ).

3.2 Formulation of the contact conditions

Recall that displacements are described by a vector function u = <u,,u$_2$ Jof the variable x - (x,,x$_2$). The traction field on the particle boundaries is denoted either as p = (p,,p$_2$), or after projections to normal and tangential directions as p = (p„,p,) • A similar result is valid for projections of displacements, u = (u_n,u_t). Assuming the "small deformation" theory, the essential contact conditions on the interface may be formulated as follows (no penetration conditions):

$$[u]_n^k = u_n^{kc} - u_n^{ka} \le 0 \quad \text{on} \quad \Gamma_C^k, \qquad (8)$$

where /jf, k = 1,...,« are boundaries between adjacent particles, «*" is the normal displacement of current element $a = c$ and $a = a$ belongs to the adjacent element, both on the current common boundary , r£, k runs numbers of all common sides of the particles, n is the number of common sides of hexagons (having exactly two adjacent particles inside the domain, one or none on the external boundary).

Let **t** * be the spring stiffness in the normal direction and k_n^k be the spring stiffness in the tangential direction on the boundary between particles with a common boundary r*. Then in the elastic region $p_n^k = k_n^k [u]_n^k$ and $p_t^k = k_t^k [u]_t^k$. Denote

$$K = (u \in V, (p_n^+)^k \ge p_n^k = k_n^k [u]_n^k,$$

$$\text{if } (p_n^+)^k \le p_n^k \quad \text{then} \quad p_n^k = 0,$$

$$k_t^k \, |[u]_t^k| \le c^k \quad \text{on} \quad \Gamma_C^k, \quad k = 1,\dots,n \;),$$

$$[u]_t^k = u_t^{kc} - u_t^{ka}.$$

where «* " is the tangential displacement on the side k, $(p_n^+)^k$ denotes the tensile strength, c^k is the shear strength, V is the set of displacements that fulfill the kinematical boundary conditions and condition (8). If p* = 0 then set K is a cone of admissible displacements satisfying the essential boundary and contact conditions. This is valid for brittle or almost brittle material (coal seam, glass). If the material exhibits elastic-plastic behavior, then the cone K is changed as:

$$K = (u \in V, (p_n^+)^k \ge p_n^k = k_n^k [u]_n^k,$$

$$\text{if } (p_n^+)^k \le p_n^k \quad \text{then} \quad p_n^k = 0$$

$$k_t^k |[u]_t^k| \leq c^k \chi(p_n^k) - p_n^k \tan\phi \text{ on } \Gamma_C^k, \quad k = 1,\ldots,n \quad),$$

$$[u]_t^k = u_t^{k,c} - u_t^{k,a}.$$

where ϕ is the angle of internal friction, and p^k_n is the normal traction on the side k, x 's me generalized Heaviside's function being equal to zero for a positive argument and equal to one otherwise. Here the sign convention is important: positive normal traction is tension.

From the above-defined spaces we can deduce that $p_n^k, [u]_n^k$, and $p_t^k, [u]_t^k$ behave linearly between certain limits, which are given by the material nature of the body.

The total energy J of the system reads:

$$J(u) = \frac{1}{2} a(u, u) - \sum_{k=1}^{n} \int_{\Gamma_C} (k_n^k ([u]_n^k)^2) d\Gamma - \int_{\Omega} b^T u d\Omega \quad (9)$$

$$a(u, u) =$$

$$= \int_{\Omega_0} \varepsilon^T C \varepsilon d\Omega_0, \varepsilon = (\frac{\partial u_n}{\partial x_n}, \frac{\partial u_t}{\partial x_t}, \frac{1}{2}(\frac{\partial u_t}{\partial x_n} + \frac{\partial u_n}{\partial x_t})).$$

where e is the strain tensor, C is the stiffness matrix of the particle, T denotes transposition,.Ω_0 is the sum of subdomains Ω, i.e., of hexagonal elements, b is the volume weight vector.

Note that the spring stiffness $k[$ plays the role of a penalty. Recall that the problem can also be formulated in terms of Lagrangian multipliers, and then leads to mixed formulation. The latter case is more suitable for a small number of boundary variables; the problem discussed in this thesis decreases the number of unknowns introducing the penalty parameters.

FORMULATION OF TFA

In this section, our aim is to formulate the general procedure for the TFA. This may be done in terms of many modem numerical methods. It seems that the BEM is the most appropriate numerical method in this case, but the FEM is also admissible.

First, let us consider that the body (part of a structure, element, system of more elements, composite) behaves linearly; i.e. Hooke's linear law is valid in entire body (this assumption admits among others an application of the BEM). When the problem is correctly posed, the displacement vector, strain and stress tensors can be obtained from the Navier equations, kinematical equations, and the linear Hooke's law.

In the second step we select points, where the measured values are available, either from experiments in laboratory, or from "in situ" measurements. We also select points, or regions (subdomains) from the body under study, and apply there successively unit eigenparameter impulses (either eigenstresses or eigenstrains) to get an influence tensor (matrix). In order to precise this statement, denote $A_{,}$, i = l,...,n, either the points or regions where the eigenparameters will be applied. Let, moreover, the set of points where the measured values are known be S„ j = l,...,m. Then the real stress at ß; is a linear hull of stress $a^{e\%}$' at Bj due to external loading and eigenstrains n and e^{p}', or eigenstress X and relaxation stress 0^{rel} at A_t (similar relations are valid for overall strain field E):

$$\sigma = \sigma^{ext} + P^{\sigma} \mu + Q^{\sigma} \varepsilon^{pl}, \quad \text{or}$$

$$\sigma = \sigma^{ext} + R^{\sigma} \lambda + T^{\sigma} \sigma^{rel}, \quad (10)$$

$$\varepsilon = \varepsilon^{ext} + P^{\varepsilon} \mu + Q^{\varepsilon} \varepsilon^{pl}, \quad \text{or}$$

$$\varepsilon = \varepsilon^{ext} + R^{\varepsilon} \lambda + T^{\varepsilon} \sigma^{rel}, \quad (11)$$

or in differential (incremental) form:

$$d\sigma = d\sigma^{ext} + P^{\sigma} d\mu + Q^{\sigma} d\varepsilon^{pl}, \quad \text{or}$$

$$d\sigma = d\sigma^{ext} + R^{\sigma} d\lambda + T^{\sigma} d\sigma^{rel} \quad (12)$$

$$d\varepsilon = d\varepsilon^{ext} + P^{\varepsilon} d\mu + Q^{\varepsilon} d\varepsilon^{pl}, \quad \text{or}$$

$$d\varepsilon = d\varepsilon^{ext} + R^{\varepsilon} d\lambda + T^{\varepsilon} d\sigma^{rel}, \quad (13)$$

where the influence tensors P, Q, and R and T may be identical (in the case of the TFA they must be identical, as they describe generalized linear Hooke's law, and $\mu = \varepsilon^{pl}, \lambda = \sigma^{rel}$), as any eigenparameter may stand for the plastic or relaxation parameter (say, eigenstrain may stand for plastic strain, which is obvious from (1)). The dimensions of σ, σ^{ext}, μ, ε^{pl} λ, and σ^{rel} are m x 6 (because of symmetric stress and strain tensors) and the dimensions of P and Q are m x 6 x n. In the classical TFA the values of |x , or ε^{pl} are calculated from minimization of variance of computed and measured stresses. It holds: λ = - C μ

The first relations in (10) and (12) describe the initial strain method while the second relations in those equations formulate the initial stress method The eigenparameters may generally stand for plenty of phenomena like change of temperature, swelling, watering, etc. This is why we could split the eigenparameters in (10) into two parts: eigenparameters themselves and the quantities connected with physically nonlinear behavior of the material.

DISTURBED STATE CONCEPT (DSC)

The idea of this theory was originally proposed by Desai, and the theory characterized behavior of overconsolidated clays. Since then, Desai and coworkers have developed and successfully applied this concept to other materials (Desai 1994).

The DSC is a unified modeling theory for mechanical behavior of material and interfaces. It allows incorporation of the internal changes on interfacial boundaries of phases (both micro- and macrolevel) and the resulting mechanism in a deforming material into the constitutive description. Initially, the material under external loading is in relative intact state (IS). Using such theories as elasticity, plasticity and viscoplasticity may theoretically treat the intact state, i.e. no cracking is considered in this state. After increasing the external loading, the material transforms from the IS state to the fully adjusted state (FA) or critical state, which is an asymptotic state, the material at that may no longer carry certain or all stresses. For example, microcracking and subsequent softening are such disturbances.

Desai uses a scalar disturbance function *D,* having different expressions depending of mechanical properties in the model under consideration. The equilibrium equation for a material element in terms of stresses is derived as:

$$D_u S_{ij} = (D_u - D)\, \sigma^{IS}_{ij} + D\, \sigma^{FA}_{ij}\ , \qquad (14)$$

where *S,,* stands for average (observed) response, D_u is max *D* and is in most cases equal to one. Using the incremental method, the differentiation of the last relation yields:

$$D_u\, dS_{ij} = (D_u - D)\, d\sigma^{IS}_{ij} + D\, d\sigma^{FA}_{ij} + dD\, (\sigma^{FA}_{ij} - \sigma^{IS}_{ij}) \qquad (15)$$

The first term of the right hand side of the last equation expresses continuum constitutive law for elastoplastic (visco-plastic) behavior, the second term obeys the classical Kachanov damage formulation (Kachanov 1992), and the third term in (15) indicates different stresses in the two parts.

The incremental constitutive equations for the IS part and the FA part are expressed as:

$$d\sigma^{IS}_{ij} = C^{IS}_{ijkl}\, \varepsilon^{IS}_{kl},\ \ d\sigma^{FA}_{ij} = C^{FA}_{ijkl}\, \varepsilon^{FA}_{kl} \qquad (16)$$

where C^{IS}_{ijkl} are in our case the components of von Mises-Huber-Hencky constitutive tensor IS part, furthermore C^{FA}_{ijkl} are the components of damage constitutive tensor for FA part, superscripts IS and FA indicate the phases. For more details concerning the DSC see Desai's publications cited in References.

For Hooke's law with Mises condition involving eigenstrain it holds in incremental form:

$$\varepsilon_{ij}(du) = A_{ijkl}\, d\sigma_{kl} + d\,(\varepsilon_{ij})^{pl} + \mu_{ij}\ , \qquad (17)$$

where (Duvant and Lions 1972):

$$(\varepsilon_{ij})^{pl} = 0 \quad \text{for} \quad F(\sigma) < 0 \quad \text{and}$$

$$(\varepsilon_{ij})^{pl} = 1/(2Gs)/(s - k)(\sigma_{ij})^D \quad \text{for} \quad F(\sigma) > 0,$$

$$2s^2 = (\sigma_{ij})^D (\sigma_{ij})^D$$

and $(\sigma_{ij})^D$ are components of deviatoric part of the stress tensor, $F(\sigma)$ is the function of plasticity, which is defined for Mises model as:

$$F(\sigma_{ij}) = ½\, (\sigma_{ij})^D\ (\sigma_{ij})^D - k^2$$

In the last formulas G is the shear modulus, k is a material positive constant.

Using the dual transformation formulated by Duvant and Lions, we arrive at the following variational principle:

$$\int_\Omega \{ \frac{1}{2} K(div\, u)^2 + \Phi(\varepsilon^D)u \} dx - \int_\Omega f_i u_i dx - \int_{\Gamma_p} p_i u_i dx$$

$$\Rightarrow \min, \qquad (18)$$

where

$$\Phi(\varepsilon^D) = G(\varepsilon_{ij})^D(\varepsilon_{ij})^D \ \text{for}\ 2G^2(\varepsilon_{ij})^D(\varepsilon_{ij})^D < k^2,$$

$$\Phi(\varepsilon^D) = k\, ((2(\varepsilon_{ij})^D(\varepsilon_{ij})^D)^{1/2} - k/2G)$$

$$\text{for } 2G^2(\varepsilon_{ij})^D(\varepsilon_{ij})^D > k^2,$$

Ω is the domain with a boundary r, r_p is the part of /"where the tractions are prescribed, f is the function of volume weight, p are tractions, $(\varepsilon_{ij})^D = (\sigma_{ij})^D/2G$ are components of the strain deviatoric tensor and K is the bulk modulus. The minimum of the functional is sought for such displacements u, which fulfil! the geometric boundary conditions.

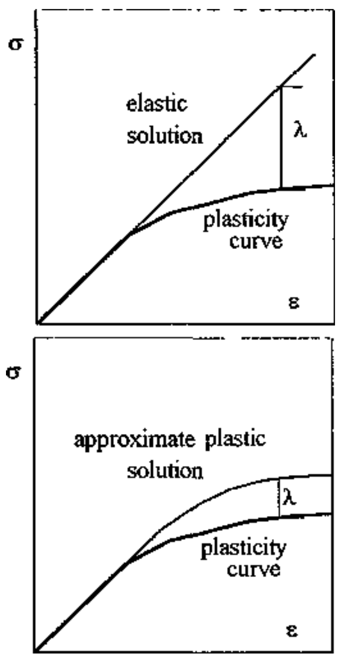

Figure 2. TFA model and unified DSC & TFA model

In order to make more transferable the TFA and the improved concept using Desai idea, a uniaxial stress-strain distribution is depicted in Figure 2. One can observe that first the elastic problem is solved, where even not too precise values of material properties are required. From the second picture in Figure 2 it is seen that nonlinear material behavior is estimated and the relaxation stresses improve the constitutive law in accordance with measurement.

TFA & DSC CONCEPT

Recall first the TFA involving the DSC. The transformation field analysis consists of expressing the stress σ at an arbitrary point ξ of the domain by virtue of superposition of stress $\sigma^{ext}(\xi)$ at ξ due to external loading which is applied to a plastic, viscoplastic, or others matrerials, and a linear hull of, say, the eigenstresses λ_l and plastic strains ε^{pl} at other points x. Since we assume that at each point six values of stress, plastic stress, and eigenstress tensors are prescribed, the relation stresses σ^k at the points S*, k = l,...,m, and the eigenstresses and plastic stresses $\lambda_1{}^l$, $l = 1,\ldots,n$ and $(\sigma^{pl})^l$, $l = 1,\ldots, m$ at *Ai* becomes (to simplify the expressions the vector notation for stress and strain tensors is used), cf. (1):

$$(\sigma_i)^k = (\sigma^{ext}{}_i)^k + \sum_{j=1}^{6} \sum_{l=1}^{m} (R^{\sigma}{}_{ij})^{kl} (\lambda_j)^l + \sum_{j=1}^{6} \sum_{l=1}^{m} (T^{\sigma}{}_{ij})^{kl}, (\sigma^{rel}{}_j)^l, \quad i = 1,\ldots,6, \; k = 1,\ldots,m, \quad (19)$$

or

$$(\sigma_i)^k = (S^{\sigma}{}_i)^k + \sum_{j=1}^{6} \sum_{l=1}^{n} (R^{\sigma}{}_{ij})^{kl} (\lambda_j)^l, \quad i = 1,\ldots,6, \; k = 1,\ldots,m, \quad (20)$$

where $(S^{\sigma}{}_i)^k$ express the current state of the overall stresses involving nonlinear changes in the material.

Note that similar relations can be written for displacements:

$$(u_i)^k = (u^{ext}{}_i)^k + \sum_{j=1}^{6} \sum_{l=1}^{m} (R^{u}{}_{ij})^{kl} (\lambda_j)^l + + \sum_{j=1}^{6} \sum_{l=1}^{m} (T^{u}{}_{ij})^{kl}, (\sigma^{rel}{}_j)^l, \quad i = 1,\ldots,6, \; k = 1,\ldots,m, \quad (21)$$

or

$$(u_i)^k = (S^{u}{}_i)^k + \sum_{j=1}^{6} \sum_{l=1}^{n} (R^{u}{}_{ij})^{kl} (\lambda_j)^l, \quad i = 1,\ldots,6, \; k = 1,\ldots,m. \quad (22)$$

On the other hand measured stresses $(\sigma_i^{meas})^k$, or measured displacements $(u^{meas}{}_i)^k$ are available in a discrete set of points. A natural requirement is that the values of measured and computed values be as close as possible. This leads us to the optimization of an "error functional"

$$I[(\lambda_j)^l] = \sum_{i=1}^{6} \sum_{k=1}^{m} [(\sigma_i)^k - (\sigma_i^{meas})^k]^2 \rightarrow \text{minimum}, \quad (23)$$

or

$$I[(\lambda_j)^l] = \sum_{i=1}^{6} \sum_{k=1}^{m} [(u_i)^k - (u_i^{meas})^k]^2 \rightarrow \text{minimum} \quad (24)$$

Differentiating / by $(\lambda_\alpha)^\beta$ yields a linear system of equations for $(\lambda_j)^l$:

$$\sum_{j=1}^{6}\sum_{l=1}^{n}(A_{\alpha j})^{\beta}(\lambda_j)^l = Y_\alpha^\beta,\ \alpha=1,...,6,\ \beta=1,...,m, \tag{25}$$

where

$$(A_{\alpha j})^{\beta} = \sum_{i=1}^{6}\sum_{k=1}^{m}(R_{ij})^{kl}(R_{i\alpha})^{k\beta},$$

$$Y_\alpha^\beta = -\sum_{i=1}^{6}\sum_{k=1}^{m}(S_i)^k - (u^{meas}{}_i)^k +$$

$$+\sum_{j=1}^{6}\sum_{l=1}^{m}(R_{ij})^{kl}(\lambda_j)^l(R_{i\alpha})^{k\beta}$$

In order to get d(o;/ in (8), one needs to calculate d$(\lambda_j)^l$. This is the difference betwe $(\lambda_j)^l$ rom (11) and the same quantities from the previous step of incremental method.

The procedure deserves a closer attention. In the first step the influence matrices are created, as described in the above explanation (Sect, on formulation of the TFA). The distribution of the disturb function *D* is determined mostly from laboratory tests. The incremental method is recommended when applying the DSC & TFA.

Let us start with some load of the trial body.

At the beginning the FA state will be most probably not reached. The Intact State follows the von Mises-Huber-Hencky law. When increasing the load, the DSC has to involve both IS and FA states into the computation. Then, the stresses are split into IS and FA parts. Increments of both these parts can be done from the DSC and the total stress for both parts is given by adding the increments to the previous steps. The same is valid for the total current stress; see (7).

Since the relations (20) and (22) are linear, substitutions of *(S,f* and $(\lambda_r)^k$ there do not change the linearity. Then applying the minimum condition for the additional eigenstresses, the improvement of current stresses or error for the DSC is obtained.

The above-described procedure can be created for measured displacements in a similar way. The displacements have to substitute overall stresses and the "error functional" has to be employed. This is not in the full compliance with classical Transformation field analysis, but it follows from mechanical point of view.

CLASSICAL CONTINUUM DAMAGE MODEL

In the previous section, λ_1 was used to express "error function", improving the choice of plasticity model. The function λ_2 will express the influence of the damage.

In the continuum damage model it is assumed that the damaged parts can carry no stress at all, and they act as voids. In other words, the observed response derives essentially from the undamaged parts; their stress-strain-strength behavior is degraded because of the existence of the damaged parts. For example, the damage parameter, ω, is defined as

$$\omega = 1 - \frac{V^V}{V}$$

where *V* is the volume of the damaged parts and *V* is the total volume of the material element. Then, $\omega =$ *D* represents the special case and appears in X_2 as an argument.

In the sense of the Unified TFA & DSC Concept the equilibrium for d *a* can be written as

$$\sigma = \sigma^{IS} - \lambda_2(\omega),$$

The representation of the function of $\lambda_2(\omega)$ can be found in (Kachanov 1992), for example.

In comparison to simple TFA & DSC, the involvement of free hexagons leads to nonlinear equations because of impossibility defining an appropriate cone. On the other hand, this approach seems to be very promising, as it offers one of the most efficient procedure leading the minimum variance of measured and computed results. The expressions (19) and (21) together with variational principle (23) enable us to connect both types of measurement.

DISCUSSION OF THE RESULTS

The material values has been selected as

Fibers: E_r = 414 GPa, v_f = 0.19, c_f = 0.25

Matrix: En, = 99.5 GPa, v_{ra} = 0.3

F™«* = 48.7 GPa, v_{mres} = 0.42

k^2 = 510Mpa

where c_f is the fiber volume fraction. Two-dimensional case is solved. Because of symmetry, overall strain has been applied in two directions: E_n*md* E_{12}- The responses in damage (debonding of matrix from fibers) and plastic zones are calculated and depicted in Figs. 4b and 4c. The debonding corresponds a typical "75° zones along the interfacial boundary while plastic zones creating tongues between fibers. They are relatively long, as the plastic zone is limited by relatively low value of k.

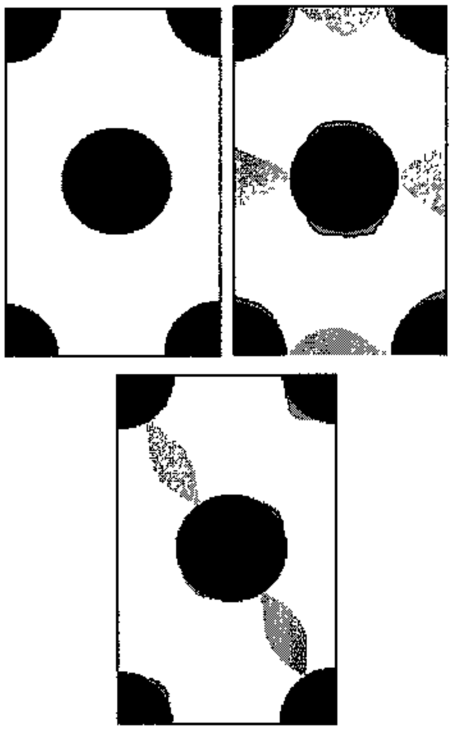

Figure 4. Damage and plastic behavior of rectangular RVE in periodic fibers: a) Geometry of the RVE, responses of b) E_n and c) E_n •

CONCLUSION

Generalized transformation field analysis is improved by Desai's DSC. In the model, damage is involved in a natural way using eigenparameters, which are known from classical TFA. They represent either plastic strains in Desai's model and play role of "error function", showing how exact the model and material parameters are selected; also describe the damage properties involved in Unified model. The results correspond the classical damage (debonding) behavior. The plastic behavior follows a concentration of stresses in the zones describing the plasticity.

4 CONCLUSIONS AND REFERENCES

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