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Solution of Ore Blending Problem by Stochastic Approach

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ABSTRACT: Ore flows from production faces, seams or orebodics may have different qualitative features. Therefore, the grade fluctuations may lead to quality variations in the finished product. One available option to deal with this problem is to blend ores coming from different sources. For this reason the blending rates should be determined from each face, seam or orebody to provide a mix that effectively achieves implicit blending. In this paper, the problem is expressed as a minimisation problem of the total material cost of the finished product such a way as to satisfy the blending requirements. Using information of the random variables that characterise the mineral contents of each ore source, chance constraints are straightforwardly converted into deterministic constraints. Then the problem is solved by the simulated annealing (SA) algorithm to find so-called best result. The variability of each variable in each flow was quantified by semi-variograms. Each flow was simulated to reproduce the characteristics, or behaviour, of the phenomenon as observed in the available data. The expected value of each variable in each flow was calculated by averaging of the simulated values. The technique is demonstrated on a case study compared with the result obtained by Zoutendijk's Method.

I INTRODUCTION

Mineral blending facility is used in various industrial operations. For instance, ore processing and metallurgical plants for ore concentration, coke ovens, blast furnaces, coal washery, copper and bauxite, pyrite-fed sulphuric acid, cement, fertiliser, palletising and glass production plants.

Regardless of the extent to which an acceptable production and homogenisation schedule is accomplished, the stockpiling may not satisfy contractual constraints or plant requirements. In the stockpiling operation raw material is added having a high- or a low-grade in a given proportion to meet the specifications (Gy. 1999). In this case the blending (proportioning) rates should be determined.

The linear programming (LP) has often been recommended as the best and the most widely applied method for blending goals (Gershon, 1988; Bott D.L. and Badiozamani K.. 1982; Gunn and Rutherford, 1990). For example, in order to meet Fe and SiO: specifications in an iron-steel plant, the blending problem can be defined as a minimisation problem in the deterministic manner:

$$Min \quad \sum_{i=1}^{n} p_i x_i$$

subject to:

$$F_L^{\bullet} \le \sum_{i=1}^{n} F_i x_i \le F_U^{\bullet}$$

$$S_L^{\bullet} \le \sum_{i=1}^{n} S_i x_i \le S_U^{\bullet}$$

$$\sum_{i=1}^{n} x_i = 1$$

$$x_i \ge 0 \quad \forall_i$$

where:

-v,	is the blending ratio for ore type /
/<	is the unit price of ore type /
Fi	is the Fe content of ore type /
S,	is the SiO; content of ore type /
FL and Fu	are the lower and upper limits for acceptable Fc content
Si anıl S_U	arc the lower and upper limits for acceptable SiO: content
	is the number of ore types or sources

However, The LP has important drawbacks:

I. The LP uses only a single goal in the objective function whereas there may be two or more objective functions in some applications. This kind of problem can be solved after extensive

modification of the LP, which is quite time-consuming (Lyu *et. ai,* 1995).

2. As the number of constraints increases, the convergence becomes increasingly difficult to achieve whereas a realistic case involves many constraints.
3. The LP might require unrealistically small amounts of some ores or leave "small heaps" of ore in the inventory which would just be a nuisance (Candler, 1991). In other words, the LP may yield extreme solutions, which cannot be used in practice. In order to avoid this problem, the modifications that incorporate additional constraints in the LP can be used (Lai and Chen, 1996).
4. The LP yields optimal results for the formulated problem but not for the real world problem because the LP takes no account of the random nature of ore variables. Traditional LP can be operated on the basis of deterministic values.

These drawbacks of LP induce to seek alternative approaches.

2 STOCHASTIC APPROACH

In the work described here an integration of Chance-Constrained Programming (CCP) (Chaînes and Cooper, 1963) and the simulated annealing (SA) (Laarhoven and Aarts, 1987; Eglese, 1990; 1992; Dowsland, 1993; Ansari and Hou, 1997) is used.

The CCP comprises 'chance constraints', which incorporate a strict measure of the probability with which the constraints must be met. For example, the chance constraints of the Fe content in an ore blend fed to an iron-steel operation may be specified as:

$$P\left[F_L^{\cdot} \le \sum_{i=1}^{n} F_i x_i\right] \ge \alpha_F \qquad \textbf{(3)}$$

where:

v,	is the blending ratio for ore type *i*
F,	is the Fe content of ore type /
F_L	is the lower limit for Fe content
n	is the number of ore types or sources
a_F	is the reliability or risk level for the constraint on Fe grade
P	is probability

The constraints require the specification of both the target qualities and the specified probability of meeting the target quality. F_L* *and ar* are deterministic values. On the other hand, F, is random variable.

Assume the random vector of Fe content for each different ore source:

$$F = (Fi.i=l.\ldots\ldots n \qquad \textbf{(4)}$$

The expected value, *E(F),* and ««variance matrix of F, *VAR(F),* are:

$$E(F) = \{E(F_i).\ i=l, \ldots\ldots, n)\} = \tilde{m}^T \qquad (5)$$

$$VAR(F) = V_F = COV(F_{,}, FJ) \qquad (6)$$

The *Fe* content of ore blend is a random variable, *ly*

$$r_F = \sum_{i=1}^{n} F_i x_i \qquad \textbf{(7)}$$

The content has an expected value and a standard deviation, which depend upon the values assigned to the non-random decision variables *x:*

$$\mu_{r_F} = \tilde{m}^T x \qquad (8)$$

$$\sigma_{r_F} = (x^T V_F\ x)^{0.5} \qquad (9)$$

where *x* is the column vector and .t^{T}is its transpose. The expected value and variability of each variable in each ore flow should be quantified. This aspect is discussed in the next section.

After the mean and variance are determined for each ore source, the distribution of the random variable, /•/-. should be specified for the reliability level. If the F,'s are normally distributed, the variate /> also exhibits normal distribution. The following variate is obtained:

$$Z(\alpha) = \frac{r_F - \mu_{r_F}}{\sigma_{r_F}} \qquad (10)$$

$$F_Z\{Z(\alpha)\} = \alpha \qquad (11)$$

where *F-A-)* is the cumulative normal distribution function. Integrating the above two equations yields

$$\Pr[r_F \le (\mu_{r_F} + Z(\alpha)\sigma_{r_F})] = \alpha \qquad (12)$$

and the deterministic equivalent is expressed as:

$$F_L^{\cdot} \le \sum_{i=1}^{n} \mu_F x_i + F_z^{-1}(\alpha_F)(x^T V_F x)^{0.5} \qquad \textbf{(13)}$$

The similar equivalents can be derived for other variables if any. The grade components of the ore sources do not necessarily follow normal distributions. Sengupta (1972) summarises how non-normal distributions can be used in CCP but the solutions become significantly more complex. Liu (1999) indicated that complex CCP models could be solved by modern heuristic methods such as neural network, genetic algorithms or simulated annealing.

2.1 Estimation of means and variances used in stochastic programming

The variance of the grades of samples within some specified volume is equal to the mean value of the semi-variogram of the samples within the specified volume. The variability of the grades over time in the ore stream determined by the sequencing model can be estimated from the semi-variogram model:

$$\gamma(h) = \frac{1}{2N(h)} \sum_{(i,j)|h_{ij}=h} (v_i - v_j)^2 \qquad (14)$$

where *h* us separation vector, *N* is the number of data pans separated by *h* and i".v are data values over the *N(h).* The dispersion variance may thus be regarded as a type of variogram calculation in which pairs of values are accepted in the averaging procedure as long as the separation vector h_v is within the ore stream, *S:*

$$\sigma^2(o/D) = \frac{1}{2N(S)} \sum_{(i,j)|h_{ij}\in S} (v_i - v_j)^2 \qquad (15)$$

where *s'(o/D)h* the variance of the average value of the attribute (e.g. grade) of sampling sizes within the total deliverable tonnage. Although this could be estimated from a set of sample data, it is usually derived from a semi-variogram model:

$$\bar{\sigma}^2(o/D) = \bar{\gamma}(S) \qquad (16)$$

where the right-hand side refers to the semi-variogram model $y< S)$ averaged over all possible vectors within *S.* In practice, the ore stream is subdivided into *n* discrete time intervals and the average semi-variogram values can be calculated by approximation of the exhaustive average of the semi-variogram by an average of the *n~* semi-variogram values at the *n* discrete time intervals:

$$\bar{\sigma}^2(o/D) = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \bar{\gamma}(h_{ij}) \qquad (17)$$

Sequential Gaussian simulation can be used to reproduce the characteristics, or behaviour, of the phenomenon as observed in the available data. Global means are calculated by averaging of the simulated values.

3 SIMULATED ANNEALING

Simulated annealing is a stochastic method for solving large objective combinatorial minimisation problems. The method is based on the principle of stochastic relaxation. Simulated annealing was developed by Kirkpatrick et. al. (1983) in the mid-80's. The method has an analogy in thermodynamics, specifically with the way that liquids freeze and crystallise or metal cools and anneals.

Suppose that a cost function in many variables is to be minimised. A simple and iterative local search could be performed to find the minimum cost. During the local search process, an initial solution is given and then a new solution is selected at random. If the cost of the new solution is lower than that of the current solution, the current solution is replaced by the new solution. Unfortunately, a local search may get stuck at local minima. Let /: $X \rightarrow R$ be a function to be minimised over *X,* where X is a finite, but very large set. A neighbourhood $N(x)$ c X is associated with each element $x \in X$. Iterations can be defined by first selecting a starting point and then repetitively selecting y e $N(\backslash)$ and comparing successive values. Simulated annealing allows the choice of v to be governed by the following stochastic rule:

$$p_{xy}(T) = \min\{1, e^{-(f(y)-f(x))/T}\} \qquad (18)$$

where;

/>„	is the acceptance probability
T	is a parameter known as temperature
A	is the current solution
y	is the new solution
$\Delta f = (f(y) - f(x))$	

As *T* approaches zero, improving choices accepted and the method reduces to a pure local search. For very large values of *T* all y solutions chosen in the neighbourhood are accepted. Any $T > 0$ allows the iteration to escape from a local minimum sooner or later. Figure I. illustrates the basic simulated annealing algorithm.

Some decisions must be made prior to implementing simulated annealing. These decisions may be classified into two groups; the first relates to the choice of a cooling schedule (generic decisions) and the second relates to problem-specific decisions. Bot-h decisions affect the speed of the algorithm. Kirkpatrick et al. (1983) remarked that convergence to a global optimum required more iterations than an exhaustive search.

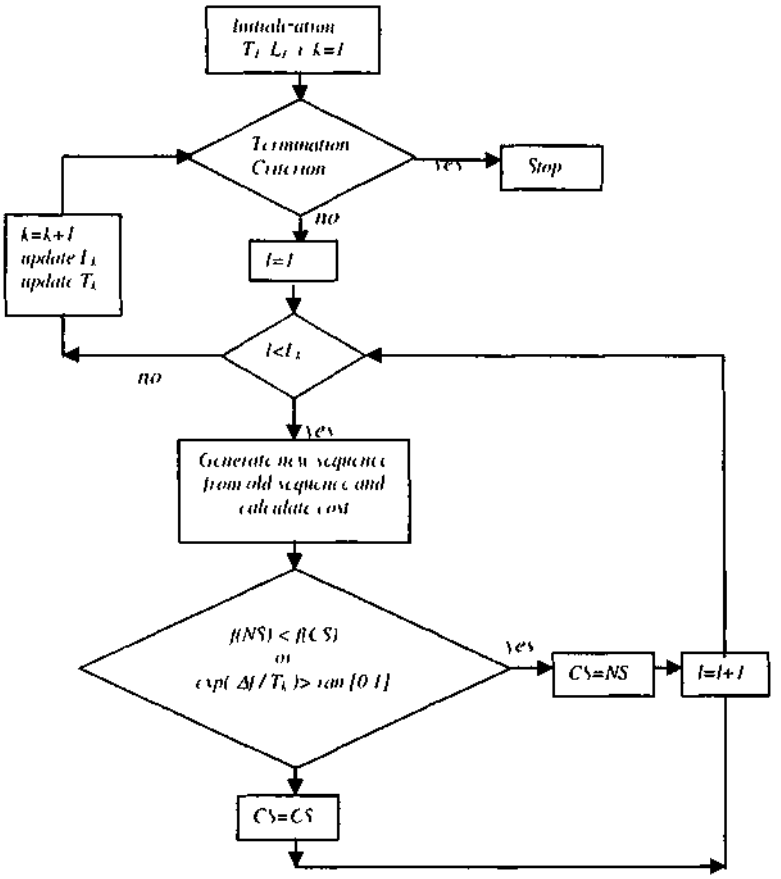

Figure 1 Basit simulated annealing algoıthm

i and j	are sequence indices
k	is the temperatuie index
T_k	is the tempeiature
l	is the iteration number
L_k	is the allowable length ot the k[th] Markov Chain
CS and *NS*	are the cunent and new proposed sequence, ıespectıvely
Rem [0,11	is a landom numbei generated between 0 and I by the i andern numbeı generatoı

4 PROBLEM DESCRIPTION

The pıoblem tıanstormed to deteıminıstıc equivalents is given as below

$$Min \quad \sum_{i=1}^{m} p_i x_i \tag{19}$$

subject to

$$\sum_{i=1}^{n} \mu_i x_i + F^{-1}(\sigma_F)\left(\sum_{i=1}^{n} \sigma_i^2 x_i^2\right)^{0.5} \geq F_U$$

$$\sum_{i=1}^{n} \mu_i x_i \geq S_U$$

$$\sum_{i=1}^{m} x_i = 1 \tag{20}$$

$$x_i \geq 0 \quad \forall_i$$

Note that this formulation includes non-linear toi m in some constraints

5 CASE STUDY

In order to demonstrate the technique, the results obtained by one of Zoutendijk's methods of feasible directions (Van de Panne and Popp, 1963) were compared with the CCP based on the SA.

Zoutendijk's method is one of available techniques for solving convex programming problems. The method is iterative and uses a feasible vector as an initial solution. The procedure iterates the non-linear problem at initial feasible solution into linear case. As non-linear functions are complicated, finding a solution is very difficult. Therefore, the technique is not practical.

The problem was cattle feed blending with probabilistic protein constraint as a minimisation problem. Same problem is often encountered in mineral industries. Therefore, one can conceive as mineral blending problem rather than cattle feed blending problem. Costs, standard deviations, protein and fat contents of raw materials to be hlended were given in Table I.

Table I. Input data

	Prolein Comeni	Fal Content	Cosl	Standard Deviation
Bailey	12.(1	2.3	24.55	0.53
Oats	11.9	5.5	26.75	0.44
Sesame Flakes	41 8	11.1	39.00	4.50
Guiundnut meal	52.1	1.3	40.50	0.79

Specified reliability level is 95% (if l-e is 0.95, q> is -1.645 from table of normal curve area). Protein commit of blend must be more than 21% and fat content of blend must be more than 5%

Minimization problem was expressed as:

$$f(x) = \sum_{i=1}^{4} c_i x_i \qquad (21)$$

subject lo:

$$12x_1 + 11.9x_2 + 41.8x_3 + 52.1x_4 - 1.645(0.53^2 x_1^2 + 0.44^2 x_2^2 + 4.50^2 x_3^2 + 0.79^2 x_4^2)^{1/2} \geq 21$$

$$2.3x_1 + 5.6x_2 + 11.1x_3 + 1.3x_4 \geq 5$$

$$x_1 + x_2 + x_3 + x_4 = 1 \qquad (22)$$
$$x_1, x_2, x_3, x_4 \geq 0$$

The minimisation problem defined above is submitted lo the SA algorithm. The computer program was written in FORTRAN 90 by the author. The required annealing parameters were specified as follows:

Initial temperature. The acceptance ratio must be almost 1 at the initial temperature and it should drop rapidly from I. In this research $T_{m,,m}i = 0.1$
Temperature decrement. $T_{J+};=a7"j$ is used as the decrement function, where a is almost I. In this research *a* is accepted as 0.95.
Stopping Criterion. The procedure is terminated when the cosl of the solution obtained in the last trial of the Markov chain remains unchanged for three consecutive temperatures.
Number of iterations in each temperature. 100 000 iterations are implemented in each temperature.
Limit of successful moves in each temperature. If 10 000 iterations are successful in any temperature, temperature is directly decreased.
Required reliability level for each temperature. Specified reliability level is 95%.

Table 2. shows results obtained from Zoutendijk's Method and CCP based on SA As seen the results, the CCP based on SA yielded very similar results to Zoutendijk's Method.

Table 2. Results compared with a traditional method

	v;	\,	11	w	,/fvi
Zoulendijk s Method	0.6359	0.0000	0.3127	0.0515	29.89 2
CCP based on SA	0.6284	0.0088	0.3096	0.0532	29.89 1

6 CONCLUSIONS

The CCP based on the SA can be easily implemented to solve the mineral blending problem. The method is flexible to changing structure of objective function and constraints. The computer running time is approximately 50 minutes. Moreover, the program can easily be incorporated into multi-objective case. The technique is able to deal with random nature of the blending problem. The performance of method was, to a large extent, depends upon selection of cooling schedule of SA. For a further research the problem can expressed as a dynamic (multi-period) approach.

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